1D Explosions

We study the explosion of a firecraker at the water surface when confined in a cylindrical tube of diameter D and depth of immersion L. The tube is vertical and opened at both ends [figure 1-(a)]. An example of explosion is presented in figure 1-(b).

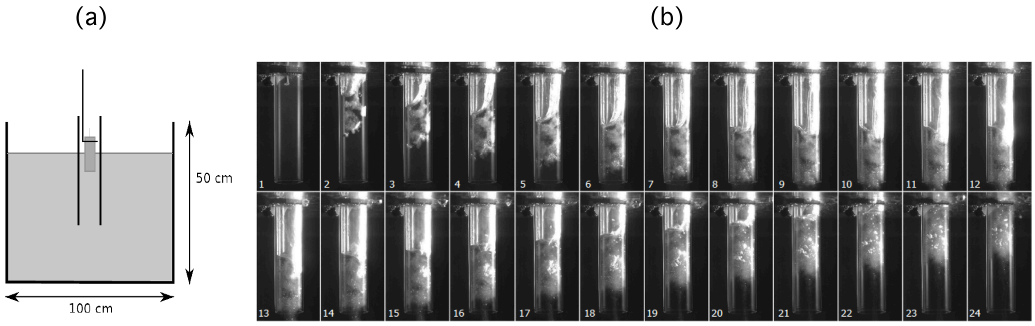

Figure 1: (a) Sketch of the expleriment (b)Chronophotography of a cavity created with a 5 gr firecracker and confined in a tube of 5 cm diameter and 18.5 cm length. The time step between images is 15 ms.

01 - Describe qualitatively the phenomenon.

02 - Evaluate the Reynolds number.

03 - Can we consider the flow as incompressible ?

04 - Can we consider the flow as irrotational ?

05 - Using the conventions presented in figure 2-(a), show that the velocity potential can be approached by:

$$\phi\,(z,t) = \dot{Z}(t).z \tag{1}$$

06 - Use the potential ϕ and the unsteady Bernoulli equation between A and B [figure 2-(a)] to get the differential equation for $Z(t)$.

07 - In the low energy limit, the maximal extension Z_{max} is small compared to L. Show that the equation of motion reduces to an oscillator equation $\ddot{Z} + \omega_0^2.Z = 0$ and determine ω_0.

08 - Integrate the equation and find the link between the maximal extension Z_{max} and the initial velocity $\dot{Z}_0$.

09 - In the large velocity limit, the equation of motion cannot be simplified. Show that it can be written in a dimensionless form as:

$$(1-\bar{Z})\,\ddot{\bar{Z}} = -\bar{Z} \tag{2}$$

10 - Integrate this equation once and find the general relation $Z_{max}(\dot{Z}_0)$. Discuss the limit $\lim\limits_{\dot{Z}_0 \to \infty} Z_{max}$.

11 - As the air cavity collapses (upgoing phase) the flow at the exit of the tube is no more quasi-parallel but behaves as a sink flow. Evaluate the difference of pressure between B and C.

12 - Deduce the differential equation for the upward motion.

13 - Comment figure 2-(b).

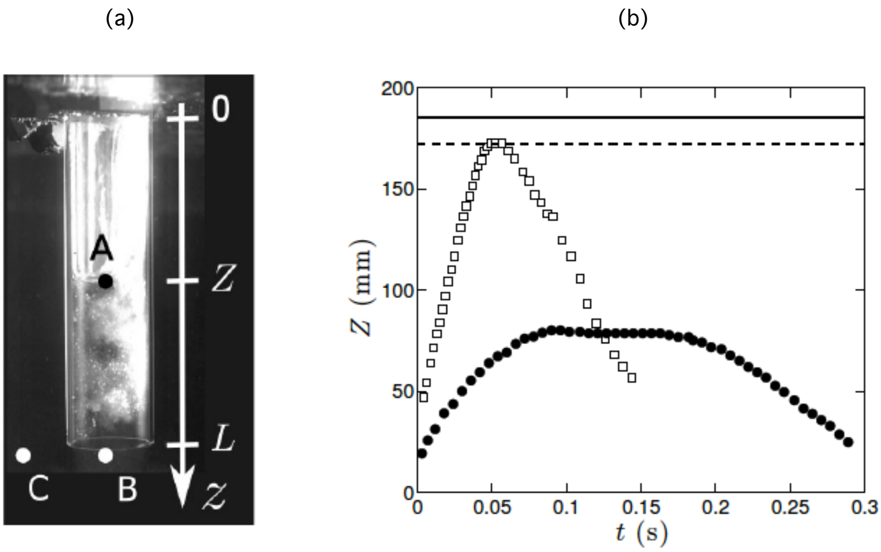

Figure 2: (a) Presentation of the conventions used for the model. (b) Size of the cavity as a function of time. In open squares, the cavity reaches the bottom end of the tube (L = 173 mm indicated with a dashed line) but does not exit ($Z \leq L$). In dots, the cavity stops far away from the end of the tube (L = 185 mm indicated with a solid line).