Nonlinear Physics

Systems governed by nonlinear equations display multiple solutions with different symmetries. We study the bifurcations i.e. the transitions between these solutions when a parameter of the system is varied. We show that in the vicinity of these bifurcations, the system is governed by universal equations, normal forms, that mostly depend on the broken symmetries at the transition. We emphasize the analogy with phase transitions, but also point out differences such that limit cycles or chaotic behaviors that do not occur at equilibrium.

1. An introduction to nonlinear phenomena.
Simple examples using electric or mechanic devices.

2. Nonlinear oscillators: quasilinear versus relaxation regimes. Frequency locking. Parametric resonance and related topics.

3. Adiabatic elimination of damped modes. Normal forms for elementary bifurcations.

4. Broken symmetries and amplitude equations.
Pattern forming instabilities in hydrodynamics. Analogy with the mean field description of second order phase transitions.

5. Broken symmetries and neutral modes. Phase dynamics.

6. Subcritical bifurcations and metastable states. Localized structures. Analogy with the liquid-vapor transition. Nucleation and Maxwell construction. Non potential effects.

M. Ben Amar

ICFP COURSE: Morphogenesis

A continuous viewpoint of morphogenesis.
Preliminaries: : This plan takes into account the advancement in the Stephan Fauve's course in the technics of nonlinear physics

1 Chemical dynamical systems and patterns formation.

1a) Chemical reactions, law of mass action, singular perturbation analysis.
1b) Mass-flux equation and reaction-diffusion equations: The Turing Instabil- ity.
1c) Mass-flux equation for biological systems: chemotaxis, haptotaxis...
Nonlinear physics tools: Boundary layer, linear stability analysis.

2 Biochemical signaling and mechanics.

2a) From cells to tissues. Introduction to elasticity and visco-elasticity.
2b) Activity or how to modify the mechanical laws.
2c) The mixture model and tumor growth.
2d) The theory of active gels.
Nonlinear physics tools : Variational techniques, self-similar solutions

3 Morphogenesis of Physical systems.

3**a)** Viscous fingering and dendritic growth: model systems for growth in physics and free-boundary problems.
3b) Volume equation and boundary conditions of Dirichlet and Neumann types.
3c) Elementary solutions: the planar and radial front solutions analysis. From viscous fingering to fractal growth.
3d) Exact soltions. Capillarity as a selection parameter. WKB methods. *Nonlinear physics tools* : Neumann boundary conditions with sources, com- plex analysis, selection

4 Free-boundary problems and living colonies.

4a) Fluid colonies of bacteria (active or passive)
4b) Moving epithelia on solid substrate: chemotaxis, durotaxis and homeosta- sis.
Nonlinear physics tools: Galilean invariance, Normal Form