# Physics of Fluids

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**Part A : wind-dispersal seeds** [1]


Figure 1: Geometrical properties of dandelion seeds (Taraxacum officinale).

The mass of a dandelion seed is 0.62 mg and its geometrical properties are presented in figure 1. The purpose here is to evaluate the size of the dispersion zone.

A1 - From your own experience, evaluate the order of magnitude of the Reynolds number which characterizes the flow around a single hair of a dandelion seed during its fall.

A2 - Deduce the expression of the drag for the whole seed.

A3 - Balancing the drag with the weight, get the expression of the terminal velocity $U_\infty$.

A4 - If $U_w$ stands for the velocity of the wind, discuss the size of the dispersion zone $L_D$, as a function of $U_\infty$ and $U_w$.

A5 - If $\gamma_a$ is the adhesion energy of the seed on the plant, determine the minimum wind speed needed to detach the seed.

A6 - Deduce $L_D(\gamma_a)$.

A7 - In the limit of geometrical similarity, where all dandelions are characterized by a single length $L$, discuss the relation between $L_D$ and $L$.

[1] From S. Minamia and A. AzumaVarious, flying modes of wind-dispersal seeds, Journal of Theoretical Biology, 225, 1-14, 2003.

## Part B: Comparative Gliding Performance in Flying Lizards [2]

Despite exhibiting considerable interspecific variation in body mass, flying lizards of the genus Draco are isometric in their area-mass scaling relationships and exhibit no significant compensatory variation in wing aspect ratio. Thus, larger species are expected to be relatively poor gliders, in lieu of behavioral or physiological compensation, when compared with smaller congeners. The purpose of this part is to discuss this point.

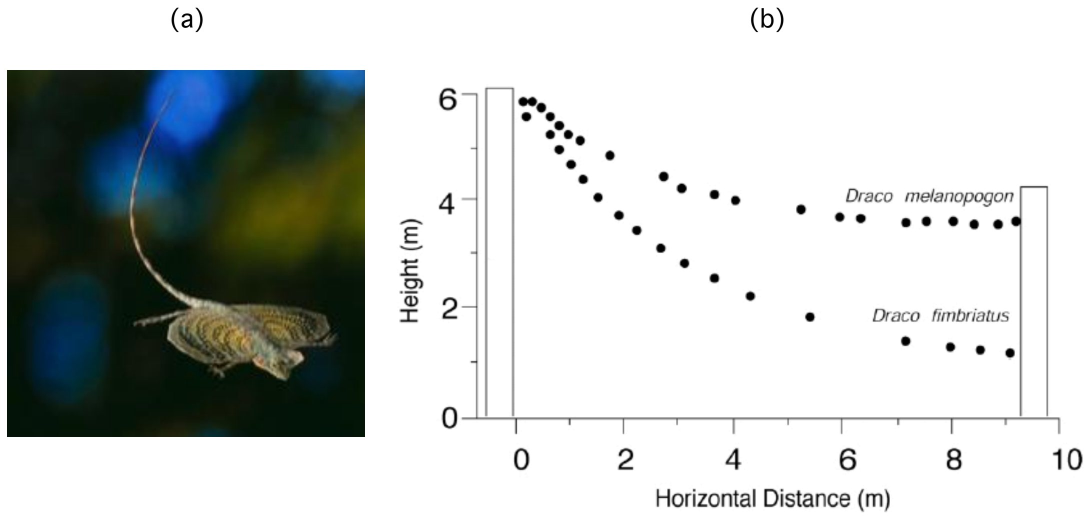


Figure 2: (a) Draco melanopogon. (b) Representative glide trajectories for two species of Draco near opposite ends of the size spectrum for the genus: Draco fimbriatus (body mass 21.6 g) and Draco melanopogon (body mass 2.95 g).

B1 - From figure 3, evaluate the Reynolds number which characterizes the flight of dracos.

| | Body mass (g) | | Wing loading ($N/m^2$) | | Adjusted glide velocity (m/s) | | Height lost (m) | | Glide angle (deg) | |
|---|---|---|---|---|---|---|---|---|---|---|
| Species | Mean | Range | Mean | Range | Mean | Range | Mean | Range | Mean | Range |
| *Draco blanfordi* ($n = 16$) | 11.3 | 3.3–16.7 | 14.1 | 9.2–17.7 | 6.4 | 5.3–7.6 | 4.7 | 3.9–5.4 | 26.6 | 22.5–30.0 |
| *D. fimbriatus* ($n = 2$) | 18.7 | 15.8–21.6 | 23.5 | 22.7–24.2 | 6.4 | 5.7–7.1 | 5.1 | 5.0–5.2 | 28.8 | 28.7–28.8 |
| *D. formosus* ($n = 13$) | 8.8 | 5.5–11.3 | 14.1 | 11.9–16.4 | 7.0 | 5.1–9.5 | 4.8 | 4.1–5.7 | 26.7 | 22.0–32.0 |
| *D. haematopogon* ($n = 9$) | 5.7 | 4.2–7.7 | 12.4 | 10.5–15.1 | 5.2 | 4.4–6.5 | 4.7 | 4.3–5.3 | 26.0 | 23.3–28.8 |
| *D. maculatus* ($n = 14$) | 4.0 | 1.7–6.3 | 12.9 | 10.4–15.1 | 5.8 | 4.9–6.8 | 4.1 | 3.8–5.2 | 23.9 | 21.8–30.0 |
| *D. maximus* ($n = 8$) | 15.6 | 5.2–32.0 | 16.0 | 11.3–22.6 | 6.5 | 5.0–7.9 | 5.0 | 4.4–5.7 | 28.8 | 21.8–30.9 |
| *D. melanopogon* ($n = 35$) | 3.8 | 1.3–5.9 | 9.2 | 7.3–12.8 | 5.5 | 3.3–6.7 | 4.1 | 2.6–6.2 | 24.0 | 15.0–32.8 |
| *D. obscurus* ($n = 3$) | 9.1 | 7.9–10.7 | 15.7 | 13.2–18.8 | 6.0 | 5.4–6.9 | 3.8 | 3.8 | 22.3 | 22.3 |
| *D. quinquefasciatus* ($n = 4$) | 6.5 | 3–8.25 | 10.5 | 9.3–11.4 | 7.3 | 5.2–10.6 | 4.1 | 3.2–5.5 | 20.6 | 18.0–23.0 |
| *D. sumatranus* ($n = 1$) | 6.2 | 6.2 | 14.8 | 14.8 | 7.6 | 7.6 | 4.0 | 4.0 | 23.0 | 23.0 |
| *D. taeniopterus* ($n = 12$) | 3.3 | 2.2–4.6 | 10.0 | 8.5–11.8 | 6.1 | 5.1–7.4 | 4.3 | 3.5–5.5 | 25.4 | 21.5–30.0 |

Figure 3: Body mass, wing loading, and glide performance data for lizards included in this study.

B2 - Write the equations of motion describing the trajectory of the draco during its flight.

B3 - From these equations, discuss the initial shape of the trajectory.

B4 - Compare to the data presented in figure 2-(b).

B5 - Using the equations, discuss the steady gliding state.

[2] from J.A. McGuire and R. Dudley, The Cost of Living Large: Comparative Gliding Performance in Flying Lizards (Agamidae: Draco), the american naturalist, vol. 166, no. 1 july 2005.

B6 - Compare to the data presented in figure 2-(b).

B7 - Determine the free fall height $H_G$ necessary to reach the steady gliding state.

B8 - Compare $H_G$ for the heaviest and lightest species.

B9 - Latest wingsuits have a drag to lift ratio (finesse) of 2.5 for a surface of 1.5 m$^2$. Deduce the heigth necessary to reach the steady gliding state for a human of 80 kg and such that $C_D \approx 0.3$.

B10 - What is then the flying speed ?

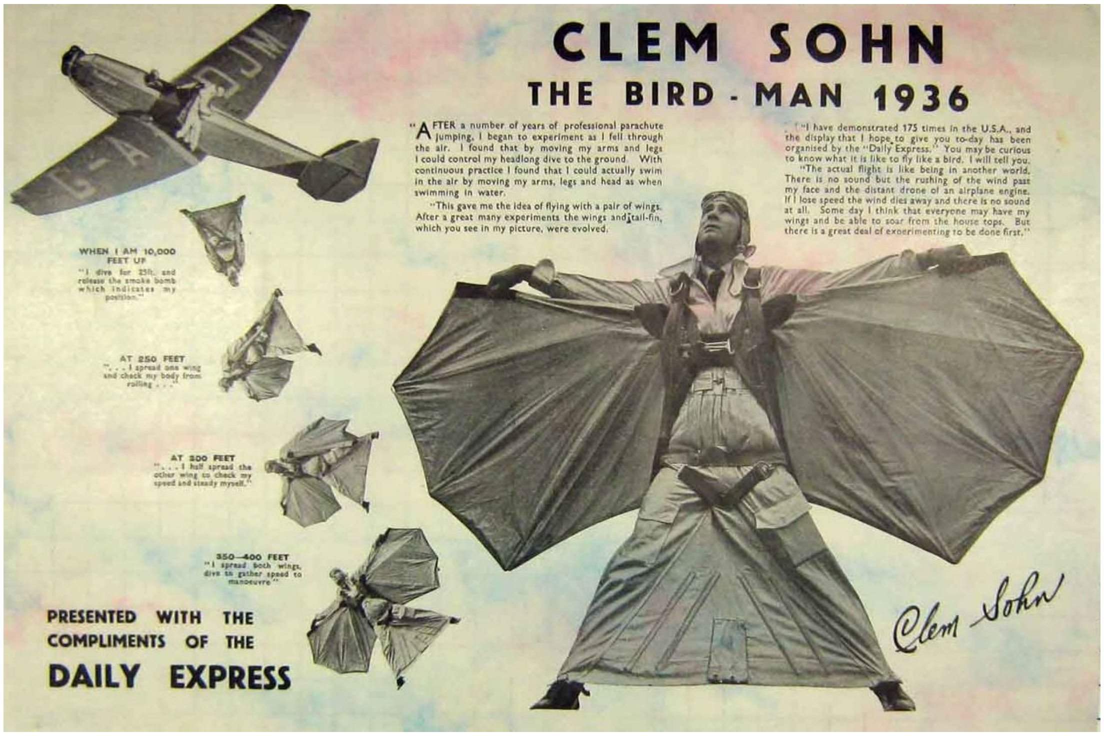


Figure 4: Présentation de Clem Sohn.

B11 - To fly, "Clem Sohn" used to jump from a plane (figure 4). Discuss the difference with base-jump.

**Part C: Wetting of nanometric hydrophobic surfaces**

In this problem, we will neglect the disjoining pressure and the long range forces. We will assume that classical capillarity is valid.

**Introduction**

C1 - A small tube of radius R is put into contact of a water reservoir. ? is the contact angle of the liquid in contact with the solid surface.
- What does small mean in the previous sentence ?
- Draw a schematic of the result of this simple experiment as a function of the contact angle $\theta$. Hydrophobic surfaces are characterized by contact angles $\theta > 90°$ and the surfaces are called hydrophilic if $\theta < 90°$. The typical length scales should be precisely drawn on the schematic.

C2 - For a hydrophobic tube of typical radius 1.3 nm, calculate the pressure needed to force the imbibition of the small hydrophobic tube for $\theta = 120°$. For a tube of length 1 $\mu m$, calculate the energy required to fill the tube.

**Nanporous materials characterization**

Instead of using a simple tube, people generally use mesoporous materials created from nanoporous silica such as micelle-templated silicas (MTSs). Such materials have quasi-1D mesopores shaped in the form of cylinders of monodisperse radius $R_p$ adjustable from 1 to 5 nm.

In order to characterize the average the nanoporous materials, it is possible to use the Langmuir adsorption isotherms. In these experiments, the porous material is put into contact with a vapor of nitrogen at 77 K. The adsorption curve is a curve of the adsorbed volume as a function of the gas pressure. The Langmuir adsorption isotherm calculation is based on the assumption that (a) only monomolecular adsorption takes place, (b) adsorption is localized, (c) the heat of adsorption is independent of surface coverage.

C3- More precisely, let V equal the equilibrium volume of gas adsorbed per unit mass of adsorbent at a pressure P and $V_m$ equal the volume of gas required to cover a unit mass of adsorbent with a complete monolayer. The velocity of adsorption depends on an activation term $e^{-E/RT}$, where E is the activation energy for adsorption.

- Justify that the velocity of adsorption is on proportional to:

$$p\left(1-\frac{V}{V_m}\right)e^{-E/RT}$$

We note E' the activation energy for desorption. The velocity of desorption is thus proportional to $kV/V_m e^{-E'/RT}$.

- Why can we write at equilibrium

$$p\left(1-\frac{V}{V_m}\right)e^{-E/RT} = kV/V_m e^{-E'/RT}$$

- Show that we can write:

$$V = V_m \frac{ap}{1+ap}$$

Where a is a constant that should be determined.
- From an experimental point of view, how will you determine the specific surface s knowing the average surface occupied by a nitrogen molecule $a_{N2} = 0,162\ nm^2$.

The typical specific surface for nanoporous materials is $s \sim$ 200–1000 $m^2/g$.

**Intrusion and extrusion measurements**
An instrumented, deformable cell is filled under vacuum with the degassed material and pure water, and water is forced in the pores up to full saturation to study the filling and drying of the hydrophobic nanopores (5). The cell volume V is then increased at a constant rate until water empties the nanopores, and the initial cell volume is recovered. The intrusion and drying transitions appear as quasiplateaus on the pressure-volume (P-V) curves.

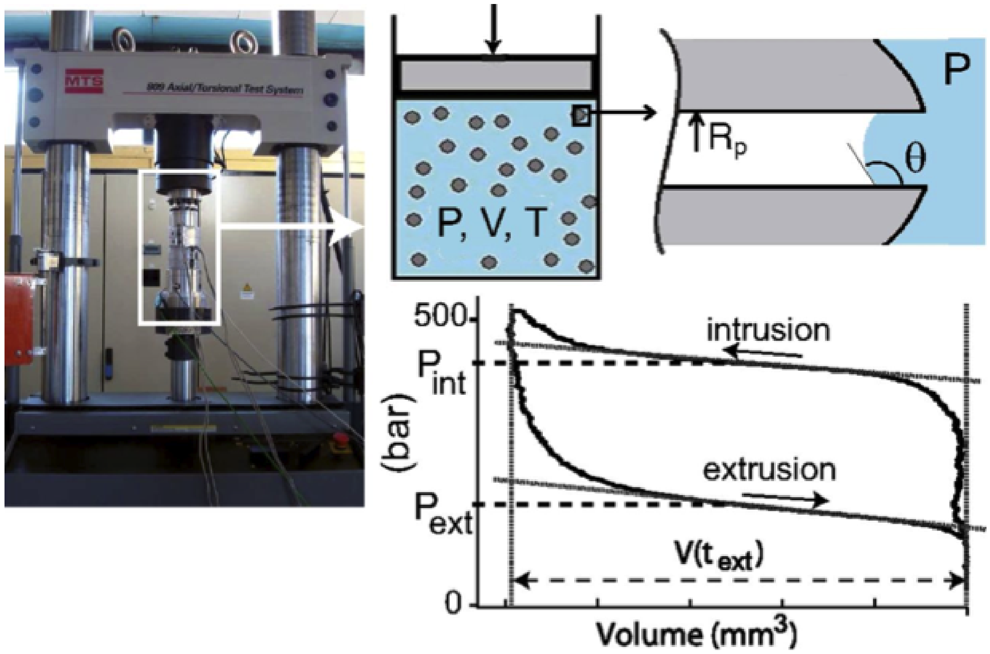


Figure 5: Intrusion/extrusion of water in hydrophobic mesoporous silicas. A thermally regulated cell containing water and the material is placed in a traction machine (Left) to measure the pressure–volume isotherms (Lower Right). The volume change is driven at a constant velocity in the range of 0.08–80 mm/s. The intrusion and extrusion pressures, $P_{int}$ and $P_{ext}$, respectively, are determined as the average pressure in the corresponding plateaus of the P-V isotherms.

C4 - Calculate the typical energy required for the imbibition of 1 g of hydrophobic nanoporous material. Discuss this value in comparison to other densities of energy.

C5 - A typical extrusion/intrusion curve is plotted in 5. The intrusion pressure $P_{int}$ is attributed to the force imbibition of the porous material.

- In order to estimate the effect of the viscous dissipation during the imbibition process, we shall calculate an order of magnitude.
- What is the relation between the pressure difference $\Delta P$ required to push a liquid of viscosity $\eta$ in a capillary of radius $R_p$ and the instantaneous injection velocity $v$ by assuming a Poiseuille flow inside the capillaries ? We note $L$ the length of the capillaries.

-If we assume that the pressure difference is kept constant during the injection process, what is the relation between $\Delta P$ and the injection time in the capillaries ?

- From the experimental data (figure 6) and an average viscosity for water $\eta = 0.4$ mPa.s, calculate the average length of the nanopores.

- Why are the curve of intrusion in 5, almost independent of the intrusion velocities ?

C6 - The extrusion pressure is assumed to be due to the nucleation of a bubble in the capillaries. Contrary to the intrusion, the extrusion is shown to depend strongly from the extrusion time (figure 7).

If we assume a rate of nucleation of the bubbles in the capillaries of the form $I = (vL/b)e^{-\Delta\Omega_c/kT}$, where $b$ and $v$ are microscopic length scale and frequency respectively, $kT$ is the thermal energy and $\Delta\Omega_c$ is the energy of the critical vapor nucleus. The nucleation occurs when the extrusion time verifies $It_{ext} \sim 1$. Dimensionally, we expect $\Delta\Omega_c = P_{ext}V_c$ with $V_c$ a critical volume.
- Show how it is possible to extract $V_c$ from the data from figure 7.
- Estimate $V_c$ from figure 7.
This measurements will allow a precise determination of $V_c$ as a function of the geometry. The model is far from the scope of this problem.

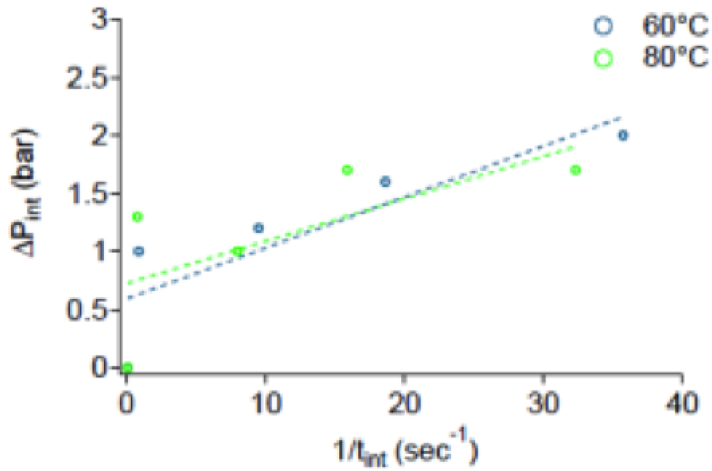


Figure 6: Injection pressure as a function of the inverse of the injection time in the capillaries, at two temperatures of injection.

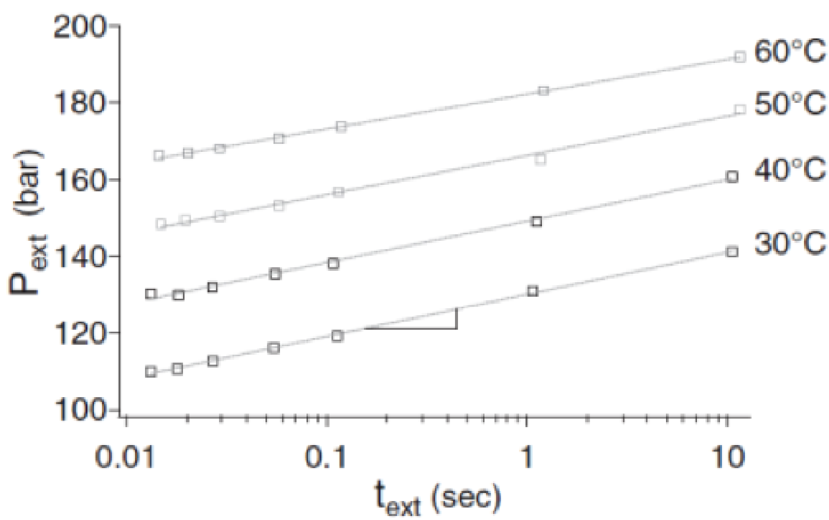


Figure 7: Variation of the extrusion pressure $P_{ext}$ with the logarithm of the time text during which extrusion occurs for the MCM-41 material at different temperatures. The other materials show similar logarithmic growth of $P_{ext}$ with $t_{ext}$.