**Quantum Dots:**

tunable puddle of electrons connected to a Fermi sea (leads)

gate (tunable)

**Kondo Physics:**

spin interacting with a degenerate Fermi sea
1. **Coulomb Blockade in Quantum Dots**
   - conductance through a nearly isolated system

2. **Kondo Effect: the basics, with quantum dots in mind**
   - localized, doubly degenerate level interacting w/ continuum

3. **Kondo Effect in Quantum Dots – selected topics**
   - Kondo box & mesoscopics, double dots, SU(4)
Coulomb Blockade in Quantum Dots

I. Overview of the Phenomena – no quantum interference effects!
   a) Geometry, 1 junction, charge steps
   b) 2 junctions
   c) Nonlinear transport

II. Magnitude of the Conductance
   a) Thermally activated regime – rate equations
   b) Inelastic cotunneling
   c) Elastic cotunneling

III. Level Quantization and Mesoscopic Effects (interference effects!)
   a) Evidence for mesoscopic and single level effects
   b) Conductance through a single level
   c) Peak height fluctuations (if there is time…)

IV. Data: Spectacular disagreement! (Kondo effect)
Main Sources

• “Low-Temperature Transport Through a Quantum Dot”, L. I. Glazman & M. Pustilnik, cond-mat/05
• Mesoscopic effects in the Coulomb Blockade, G. Usaj, D. Ullmo, HUB; Phys. Rev. B 64, 201319(R); 64, 245324; 66, 155333; 67, 121308(R) (2001-2003).
Geometry and Charging

\[ E = \frac{e^2}{2C} (Q - Q_{\text{neutral}})^2 \]

\[ Q_{\text{neutral}} = C_g U / e \]
Grain connected to one tunnel junction and one "gate" capacitor

Average charge increases in **steps** as a function of voltage applied to the capacitor

How accurately is the charge quantized?
Is conduction through the grain possible?

[From Devoret, Esteve, & Urbina, 95]
2 Junctions: Conduction!

V drives current
U sets the neutrality point
(gate voltage)

Set U so that states (1) and (0) have the same energy
→ current flows!

[From Devoret, Esteve, & Urbina, 95]
Conduction Through a CB Island (continued)

- weakly coupled leads: $Q$ jumps sharply
- $Q$ jumps and $G$ has peak when:

$$E = \frac{e^2}{2C}(Q - Q_{\text{neutral}})^2$$

$$E_{gr}(n) - \frac{C_g}{C}enV_g^* = E_{gr}(n + 1) - \frac{C_g}{C}e(n + 1)V_g^*$$
Lateral Quantum Dots

GaAs $\text{Al}_x\text{Ga}_{1-x}\text{As}$

(a) Lateral Quantum Dot

$V_{sd}$ $V_g$

$V_{gL}$ $V_{gC}$ $V_{gR}$

$1 \mu m$

[C. Marcus group, Harvard]
Vertical Quantum Dots

Data: Lateral Quantum Dot

(a)
Dot 1
B = 30 mT
T ≈ 100 mK

[C. Marcus group, Harvard]
Data: Vertical Quantum Dot

[Data from the Tarucha & Kouwenhoven groups, NTT & Delft]
Coulomb Blockade: Non-linear Transport

(a) Grain in which \( R_1, C_1 \) dominate. Note **steps in I** as more electrons can be localized on the grain. **Coulomb Staircase**

(b) Granular film – many grains with average \( C \) and \( R \)
First observations and discussion of CB in 50’s and 60’s (Giaver & Jaklevic)

[From Glazman, 00]
Non-linear CB: Data in metal grain

Characteristic diamond-shaped region of CB

[D. Ralph, et al. ’97]
Non-linear CB: Data in semiconductor quantum dot

dI/dV plotted vs. $V_g$ and $V_{sd}$

[Tarucha and Kouwenhoven groups, '01]
A Model Dot

Time to become quantitative!! so we need a model...

\[ H = H_{\text{leads}} + H_{\text{dot}} + H_{\text{tunneling}} \]

\[ H_{\text{dot}} = \sum_{n,s} \epsilon_n d_{n,s}^\dagger d_{n,s} + E_C (\hat{N} - N_0)^2 \]

\[ H_{\text{leads}} = \sum_{\alpha k s} \xi_k c_{\alpha k s}^\dagger c_{\alpha k s} \]

\[ H_{\text{tunneling}} = \sum_{\alpha k n s} t_{\alpha n} c_{\alpha k s}^\dagger d_{n,s} + \text{H.c.} \]

- \( n \) labels states in the dot (discrete)
- \( k \) labels states in the leads (continuous)
- \( \alpha = L, R \) (ie. Left or Right)
- \( s \) labels spin

\[ \hat{N} = \sum_{n,s} d_{n,s}^\dagger d_{n,s} \] is an operator

- \( t_{\alpha n} \) is independent of \( k \)
- point contact geometry
What is the Magnitude of the Conductance?

Start at high $T$ and consider successively lower-$T$ regimes.

Highest $T$: $T >> E_C$

Charging energy is negligible compared to thermal excitation
$ightarrow$ discreteness of electron charge irrelevant

Add resistance of the two junctions in series:

$$G = \frac{G_L G_R}{G_L + G_R} \equiv G_\infty$$

1st Intermediate regime: $T \leq E_C$

In addition, assume:

1. No quantum mechanical interference effects
2. Rapid thermalization of electrons in the dot – incoherent transport (ie. inelastic relaxation rate > escape rate)
3. Resistance of each junction is > $e^2/h$

$\rightarrow$ Sequential tunneling regime, treat with rate equations

[Following Glazman `00 and Pustilnik & Glazman `04]
1. Neglect quantum interference $\rightarrow$ neglect single particle quantization
   (discussed later!)

   $T$ smears density of states $\rightarrow$ treat spectrum as continuous with DOS $\nu$

3. Need for $G_L, G_R \ll e^2/h$ – heuristic argument:
   
   Tunneling through barriers
   $\rightarrow$ Quantum fluctuations of the number of electrons
   $\rightarrow$ Should be small over time of measurement

   $$\Delta t = RC = C / G_{L,R}$$

   Use time-energy uncertainty relation:
   $$\Delta E \Delta t = \left(\frac{e^2}{C}\right)(C / G) > h$$

   $$G < \frac{e^2}{h}$$

   Supported by detailed calculation as well…
For simplicity, consider $N_0$ close to the degeneracy point $N_0^*=1/2$ 
→ consider only the charge states 0 and 1

Electrostatic energy difference:

$$E_+(N_0) = E_{1} - E_{0} = 2E_C(N_0^* - N_0)$$

Current from lead $\alpha$ into the dot via Fermi’s golden rule:

$$I_\alpha = e \left( \frac{2\pi}{\hbar} \right) \sum_{k,n,s} \left| t_{\alpha n} \right|^2 \delta(\xi_k + eV_\alpha - \epsilon_n - E_+) \times \left\{ P_0 f(\xi_k)[1 - f(\epsilon_n)] - P_1 f(\epsilon_n)[1 - f(\xi_k)] \right\}$$

$P_i$: probability for the dot to be in charge state $i$

$f(\epsilon)$: Fermi function  \(\textbf{not} \) perturbed – remember point 2 above!

Convert sums to integrals over continuous density of states:

$$I_\alpha = \frac{G_\alpha}{e} \left[ P_0 F(E_+ - eV_\alpha) - P_1 F(eV_\alpha - E_+) \right]$$

$$G_\alpha = 4\pi^2 \frac{e^2}{\hbar} \nu_\alpha \nu_{\text{dot}} |t|^2$$

$$F(\omega) = \frac{\omega}{e^{\omega/T} - 1}$$
Two additional conditions: \( I_L = -I_R \quad P_0 + P_1 = 1 \)

Solution yields for the linear conductance:

\[
G = \lim_{V \to 0} \frac{dI}{dV} = G_\infty \frac{E_C (N_0 - N_0^*)/T}{\sinh[2E_C (N_0 - N_0^*)/T]}
\]

Note:
1. All peaks have same height!
2. Asymptote 0.5 at low temperature – because of correlation

(Can carry out for multiple charge state and non-linear behavior.)

[Following Glazman ’00, and Pustilnik & Glazman ’04]
Beyond First Order: Inelastic Cotunneling

Above: lowest order in $|t|^2$, involved tunneling via a real state in the dot

Result: exponential suppression of $G$ in the CB valley because probability of a thermal fluctuation to allow occupancy is tiny, $\exp(-E_C/T)$

But, have to consider higher-order activationless processes

$\rightarrow$ tunneling via virtual states in the dot

$$A_{n,m} = \frac{t^*_{Ln} t_{Rm}}{E_C}$$

[Following Glazman ‘00]
Inelastic Cotunneling (cont.)

Final states for the many processes are different $\rightarrow$ sum **probabilities**!

$$G_{in} \approx \frac{e^2}{h} \nu_L \nu_R \sum_{n,m} \left| A_{n,m} \right|^2$$

1. Convert sum on $n,m$ to integral $\rightarrow$ introduces $\nu_{dot}^2 \rightarrow G_L G_R$
2. How many pairs of states in the dot contribute??

Phase space argument familiar from Fermi liquid theory:

Incoming energy is $\sim T$

There are $\sim T^2$ ways to divide that energy among the dot states

$$G_{in} = \frac{2}{3} \frac{G_L G_R}{e^2/h} \left( \frac{T}{E_C} \right)^2$$

By comparing with above, inelastic cotunneling dominates thermally activated transport when

$$T \lesssim T_{in} \equiv E_C \left[ \ln \left( \frac{e^2/\hbar}{G_1 + G_2} \right) \right]^{-1}$$

[Following Glazman `00]
Elastic Cotunneling

What if temperature is really very small, i.e. very few inelastic processes? Consider elastic processes:

Final states for the many processes are the same → sum amplitudes!

\[ A_{el} = \sum_{n} t_{Ln}^* t_{Rn} \frac{\text{sign}(\epsilon_n)}{E_C + |\epsilon_n|} \]

Note: electron-like and hole-like processes have different sign

[Following Glazman `00, and Pustilnik & Glazman `04]
Elastic Cotunneling (cont.)

\[ G_{el} \approx \frac{e^2}{\hbar} \nu_L \nu_R \left| \sum_n t_{Ln}^* t_{Rn} \frac{\text{sign}(\varepsilon_n)}{E_C + |\varepsilon_n|} \right|^2 \]

1. Keep only terms in which phase cancels exactly (ie. diagonal approximation – we may come back to this later…)
2. Denominator cuts off sum when \( \varepsilon_n \approx E_C \)

\[ G_{in} \sim \frac{e^2}{\hbar} \nu_L \nu_R \sum_{n, |\varepsilon_n|<E_C} \left| t_{Ln} \right|^2 \left| t_{Rn} \right|^2 \frac{1}{E_C^2} \]

a) One sum → only one factor of \( \nu_{\text{dot}} \)
   → multiply by \( \nu_{\text{dot}} / \nu_{\text{dot}} \) in order to arrive at form \( G_L G_R \)

b) Number of terms in sum introduces factor of \( E_C \)

\[ G_{el} = \frac{1}{4\pi^2} \frac{G_L G_R}{e^2 / \hbar} \frac{1}{\nu_{\text{dot}} E_C} \left( \frac{1}{N_0 - N_0^*} + \frac{1}{N_0^* + 1 - N_0} \right) \]
Elastic Cotunneling (cont.)

Compare magnitudes of elastic and inelastic cotunneling:

Elastic cotunneling dominates already when

\[ T \ll T_{el} \equiv \frac{E_C}{\sqrt{\nu_{\text{dot}}} E_C} \approx \sqrt{E_C \Delta} \]

where \( \Delta \) is the single-particle level spacing.

We’ve studied three regimes of transport so far:

- Elastic cotunneling in valley
- Inelastic cotunneling in valley
- Sequential tunneling: discrete charges
- Continuous flow

[Following Glazman ‘00, and Pustilnik & Glazman ‘04]
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IV. Data: Spectacular disagreement! (Kondo effect)
Evidence for single-particle and/or mesoscopic effects in CB

Variation in peak heights

[C. Marcus group, Harvard]
Height vs T

Peak Height Varies with Temperature!

peak height $\sim$ constant

peak height increases at lower temperature!!

[Foxman, et al.; Kastner group, MIT `94]
Peak Height Varies with Magnetic Field!

Bottom: conductance in grey for 1 CB peak as a function of B and Vg
Top: Gpeak as a function of B (along line highlighted in bottom)

Sensitivity to weak magnetic field $\rightarrow$ Interference effect!

[Folk, et al.; Marcus group, Stanford '96]
Non-linear Transport Probes Single Particle States

Apply a bias voltage between the two leads connected to the dot:

\[ \delta E \]

E\(_F\) at V=0.

L lead  \hspace{1cm} \text{dot} \hspace{1cm} R lead

Can resolve discrete states if \( \delta E >> kT \)

dI/dV

[After D. Ralph]
Discrete States in Metallic Nanoparticles

peaks caused by individual levels

[D. Ralph, et al., Harvard `97]
Discrete States in Semiconductor Quantum Dots

CB peaks vs $V_g$ for 3 different $V_{sd}$

[Johnson, et al., Delft '92] [Tarucha & Kouwenhoven groups]
Energy Scales

**$E_C$: the charging energy**

**$\Delta$: the mean single-particle level spacing**

Estimate the ratio:

$$\Delta \approx \frac{2}{\nu L^d} \quad E_C \sim \frac{e^2}{\kappa L}$$

(2 for spin; in $d$ dimensions; $\kappa$ is dielectric constant)

Using $\nu \sim n/E_F$,

$$\frac{\Delta}{E_C} \sim \frac{\kappa \hbar v_F}{e^2} \left( \frac{\lambda_F}{L} \right)^{d-1} \sim \frac{1}{r_s} \left( \frac{\lambda_F}{L} \right)^{d-1}$$

($r_s$ is electron gas interaction param.)

→ Level spacing is small compared to charging energy

(except for 1d which, as always, is special)
\( \Gamma \): the width of the single-particle levels

\( \Gamma \) is related to the conductance of the tunnel junction:
- Consider a grain attached by a single junction to a lead
- Suddenly apply a bias \( V \rightarrow eV/\Delta \) occupied levels cross the Fermi level
- Escape time, \( \tau_{\text{esc}} \), gives the width of the level: \( \tau_{\text{esc}} \sim \frac{\hbar}{\Gamma} \)

\[
I \sim \frac{2e}{\tau_{\text{esc}}} \frac{eV}{\Delta} \approx 4\pi \frac{e^2}{h} \frac{\Gamma}{\Delta} V
\]

\[
G_\alpha = 4\pi \frac{e^2}{h} \frac{\Gamma_\alpha}{\Delta}
\]

\( E_{\text{th}} \): \( \sim \) time for particle to explore the whole system

ballistic: \( E_{\text{th}} \sim \hbar v_F / L \)

diffusive: \( E_{\text{th}} \sim \hbar D / L^2 \)

\[
\frac{E_{\text{th}}}{\Delta} \equiv g \propto \frac{1}{h} \ \rightarrow \ \text{a semiclassical parameter}
\]

\[
\frac{E_{\text{th}}}{\Delta} \approx \frac{1}{4} \sqrt{N}
\]

(ballistic 2d)
Coulomb Blockade: Classical vs. Quantum

Classical:
- position: constant spacing $= \frac{e^2}{C}$
- peak height: constant given by series resistors $G_{\text{peak}} = \frac{G_1 G_2}{G_1 + G_2} \cdot \frac{1}{2}$

Quantum:
- position: single particle energies and residual interactions $\rightarrow \text{spacing fluctuations}$
- peak height: coupling from quantum state $\psi$ in dot to lead $\rightarrow \text{height fluctuations}$
CB Conductance Through a Single Level: Peak

Previously, used a continuous $v_{\text{dot}} \to$ requires $T > \Delta$.

Now consider: \[ \Gamma_\alpha < T < \Delta \] thermally activated transport (for peak!)

Again, close to the degeneracy point $N_0^* = 1/2$, consider only 2 charge states

Electrostatic energy difference: \[ E_+(N_0) = E_{|1\rangle} - E_{|0\rangle} = 2E_C(N_0^* - N_0) \]

Use rate equations for 1 level in dot! Have to keep track of spin now – $P_s$

\[
I_{\alpha s} = e \frac{2\pi}{\hbar} |t_{\alpha 0}^2| \sum_k \delta (\xi_k + eV_\alpha - \epsilon_0 - E_+) \left\{ P_0 f(\xi_k) - P_s [1 - f(\xi_k)] \right\}
\]

Convert sum to integration (note: sum only over lead states!):

\[
I_{\alpha s} = \frac{2e}{\hbar} \Gamma_{\alpha 0} \left\{ P_0 f(E_+ - eV_\alpha) - P_s f(eV_\alpha - E_+) \right\}
\]

Use: \[ I_{Ls} = -I_{Rs} = I/2 \]

\[
G = \frac{4e^2}{\hbar} \frac{\Gamma_{L0}}{\Gamma_{L0} + \Gamma_{R0}} \left[ \frac{-df/d\omega}{1 + f(\omega)} \right]_{\omega = E_+(N_0)}
\]

[Following Pustilnik & Glazman,’04]
CB Conductance Through a Single Level (cont.)

Comments:

1. The peak maximum is **not** at $N_0 = N_0^*$, but rather off by $\sim T/E_C$
2. The lineshape is asymmetric
3. Peak height is approximately

$$G_{\text{peak}} \sim \frac{e^2}{\hbar} \frac{\Gamma_{L0} \Gamma_{R0}}{\Gamma_L + \Gamma_R} \frac{1}{T}$$

note the similarity to classical height with $G \rightarrow \Gamma$

[Beenakker, van Houten, Staring]

[Beenakker, '91]
Peak Height Varies with Temperature!

[Graph showing peak height varies with temperature]

- Peak height is approximately constant.
- Peak height increases at lower temperature.

Peak Height Statistics

\[ G_{\text{peak}} \sim \frac{e^2}{\hbar} \frac{\Gamma_{L0} \Gamma_{R0}}{\Gamma_{L0} + \Gamma_{R0}} \frac{1}{T} \]

What do we know about the \( \Gamma_{L,R0} \)??

The width of each level will be different depending on how it’s wave function overlaps with the states in the leads.

Point contact connection between lead and dot: tunnel from lead \( \alpha \) to point \( r_\alpha \) in the dot

\[ t_{\alpha,n} \propto \phi_n(\hat{r}_\alpha) \]

\[ \Gamma_{\alpha,n} = \pi \nu_\alpha \left| t_{\alpha,n} \right|^2 \propto \left| \phi_n(\hat{r}_\alpha) \right|^2 \]

→ Need to know about the wave functions (and levels) in the dot

Mesoscopic perspective: instead of trying to find \( G_{\text{peak}} \) for a particular precise situation, let’s look at the statistics of \( G_{\text{peak}} \).
One of the (two) big ideas from Quantum Chaos:  

**The quantum properties of a classically chaotic system are statistically universal and the same as those of a random matrix.**

[the Bohigas-Giannoni-Schmidt conjecture, `84]

Assume quantum dot or nanoparticle is **irregular** (ballistic or diffusive)  
→ motion classically chaotic → use RMT for quantum properties

**Gaussian ensembles for the Hamiltonian:**  
\[ P(H) \propto e^{-\frac{(\beta/2)a^2}{\beta} \text{Tr} H^2} \]  
\[ \overline{H_{ij}}=0; \quad \overline{H_{ij}H_{kl}} = \frac{a^2}{2\beta} g_{ij,kl}^{(\beta)}; \]

\[ \beta=1, \quad dH=\prod_{i\leq j}dH_{ij} \quad g_{ij,kl}^{(\beta=1)} = \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}; \quad g_{ij,kl}^{(\beta=2)} = 2\delta_{il}\delta_{jk} \]

\[ \beta=1 \text{ real symmetric matrix, time-reversal symmetry – GOE} \]
\[ \beta=2 \text{ complex hermitian matrix, broken time-reversal symmetry – GUE} \]

**Caution:** some properties are system specific; use RMT for those that are independent of ensemble and so expected to be universal.
Interlude: RMT properties

Energy level statistics: lots of correlations!

Close levels repel

\[ \{ \mathcal{E}_n \} \quad s \equiv \mathcal{E}_{n+1} - \mathcal{E}_n \quad P(s) \approx \begin{cases} \frac{\pi}{2} s e^{-\left(\frac{\pi}{4}\right)s^2} & \text{(GOE)} \\ \frac{32}{\pi^2} s^2 e^{-\left(\frac{4}{\pi}\right)s^2} & \text{(GUE)} \end{cases} \]

(Wigner surmise)

Wave function (eigenvector) statistics:

1. Different eigenvectors are uncorrelated as \( N \to \infty \).

2. Components of the same eigenvector are uniformly distributed under the constraint of normalization:

\[
P(\psi_1, \psi_2, \ldots, \psi_N) \propto \delta \left( \sum_i |\psi_i|^2 - 1 \right) \quad \mathcal{D}[\psi] = \prod_{i=1}^N d\psi_i \quad \text{for } \beta = 1
\]

By integration, find the distribution of

\[ \gamma \equiv \frac{|\psi_i|^2}{\langle |\psi_i|^2 \rangle} \quad P(\gamma) = \begin{cases} \frac{e^{-\gamma/2}}{\sqrt{2\pi\gamma}} , & \beta = 1 \\ e^{-\gamma} , & \beta = 2 \end{cases} \]

(Porter-Thomas distribution)
Interlude: RMT in chaotic systems

Nearest neighbor spacing distribution in a chaotic system

Solid: RMT Wigner surmise
Dashed: Poisson statistics (random)

Expect to be valid for levels within a window of size $E_{th}$

(Bohigas, Giannoni, and Schmidt, `84)
Interlude: RMT in diffusive systems

Disordered rectangular quantum dot (onsite disorder – Anderson model)

\[ \text{Anderson Model} \quad \text{Wigner-Dyson} \]

\[ \text{Porter-Thomas} \]

\[ W = 0.2 \]

\[ W = 0.5 \]

\[ W = 1.2 \]

[Miller, Ullmo, HUB, `05]

(caution: doesn’t work well for all quantities…)

\[ B = 0 \]

\[ B = 6 \]
**Interlude: Spatial Correlation of \( \psi \) – beyond RMT**

Need to know correlation of wave function in position space (interactions!)

RMT: each element \( \psi_i \) is independent (except for normalization)

\[ \Rightarrow \text{Need to go beyond RMT: Random Plane Waves} \]

\[
\psi(\vec{r}) = \frac{1}{\sqrt{N}} \sum_{\alpha=1}^{N} a_\alpha e^{ik\hat{n}_\alpha \cdot \vec{r}}
\]

Distribution of \( \hat{n}_\alpha \) and \( a_\alpha \) are uniform and independent

\[ \langle \psi^*(\vec{r}) \psi(\vec{r}') \rangle = \frac{1}{N} \sum_{\alpha=1}^{N} \left| a_\alpha \right|^2 e^{ik\hat{n}_\alpha \cdot (\vec{r} - \vec{r}')} \]

\[ = \frac{1}{A} \left\langle e^{ik\hat{n}_\alpha \cdot (\vec{r} - \vec{r}')} \right\rangle_{\text{Fermi Surf.}} \equiv \frac{1}{A} F(|\vec{r} - \vec{r}'|) \]

\[
F(|\vec{r} - \vec{r}'|) = \frac{2d}{k_F} J_0 \left( k_F |\vec{r} - \vec{r}'| \right)
\]

\[
A^2 \left\langle |\psi(\vec{r})|^2 |\psi(\vec{r}')|^2 \right\rangle
\]
Peak Height Statistics (cont.)

\[ G_{\text{peak}} \sim \frac{e^2}{\hbar} \frac{\Gamma_{L0} \Gamma_{R0}}{\Gamma_{L0} + \Gamma_{R0}} \frac{1}{T} \]

\[ \Gamma_{\alpha,n} \propto |\phi_n(r_\alpha)|^2 \]

We know the distribution of \( \Gamma \), ie. Porter-Thomas distribution

**Assume** \( \Gamma \) at the two contacts are independent

→ Distribution of \( G_{\text{peak}} \) follows from integration

extract the dimensionless fluctuating part:

\[ \alpha \equiv \frac{\Gamma_{L0} \Gamma_{R0}}{\langle \Gamma \rangle (\Gamma_{L0} + \Gamma_{R0})} \]

\[ P_{(B=0)}(\alpha) = \sqrt{\frac{2}{\pi \alpha}} \ e^{-2\alpha} \]

\[ P_{(B \neq 0)}(\alpha) = 4\alpha \left[ K_0(2\alpha) + K_1(2\alpha) \right] e^{-2\alpha} \]

Can generalize to non-point contacts, multiple channels, effects of periodic orbits, temperature…

[Jalabert, Stone, and Alhassid, `92]
Plot of the distribution, compared to numerics for a ballistic billiard (the Robnik billiard)

[Bruus & Stone, ’94]

Experiment: CB conductance peaks in a semiconductor quantum dot, 2 temperatures

Note how 3 peaks **vanish** at the lowest T – no coupling!!

[Chang, et al. ’96]
Peak Height Statistics (cont.): Experiments

Simplest theory works well!

More detailed study reveals a few problems… but in quite good shape.

[Chang, et al. ‘96]

[Marcus, et al. ‘96]
Behavior in “odd” valleys contradicts Coulomb blockade!

[Kouwenhoven group]
Nonlinear I-V in the Kondo Regime

[Kouwenhoven group, '98]
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THE END
Title