Mainly electronic properties
Very little about mechanical/structural and optical properties
No magnetic field (cf. Mark Goerbig's lecture)

Outline

**Lecture #1**: Introduction to electrons in graphene
**Exercice session**: Klein tunnel effect in graphene
**Lecture #2**: Electric transport in graphene
Lecture #1: Introduction to electrons in graphene

Outline

I) Introduction
II) Short history
III) Experimental techniques
IV) Atomic structure
V) Band structure
VI) Low energy effective theory
VII) “Relativistic” quantum mechanics in graphene
VIII) The minimum about el-el interactions
IX) Conclusion

Cargèse, October 2008
I) Introduction: main ideas

Graphene = 2D honeycomb carbon crystal (thickness of a single atom)

Graphene’s magic comes from its peculiar band structure:
- The valence (VB) and conduction bands (CB) meet in 2 (not 6) points (K,K’) in reciprocal space [Dirac points].
- The dispersion relation close to the Dirac points is linear [diabolo] and it has electron-hole symmetry [charge conjugaison].
- The VB is full, the CB is empty: the Fermi level is right at the Dirac points.
→ Graphene is a 2 valleys (K,K’) 2D gapless semiconductor. Electrons are massless and chiral.
II) Short history
II) Pre-history

Experiments
Graphite: light bulbs, nuclear moderator,..

1962 HOPG (graphite monocrystal) [Ubbelohde]

1960-1980 Graphite intercalation compounds

1985 Fullerenes [Kroto, Curl, Smalley]

1991-1993 Carbon nanotubes [Iijima]

II) Modern history

Experiments
2004 Contacted (and gated) graphene on amorphous SiO$_2$ substrate [Novoselov, Geim]

2004 Epitaxial “graphene” on SiC [Berger, de Heer]
2005 Graphene quantum Hall effect [N., G., Zhang, Kim]

2006 Graphene bilayer QHE [N., G., McCann, Falko]
2007 Suspended graphene [Meyer]
10/2008: about 1000 preprints in the arXiv since QHE

See M. Goerbig's lectures
II) History: theories

1947 Graphene band structure [Wallace]

1956 Graphene Landau levels [McClure]

1985 Hofstadter butterfly [Rammal]

1984-1988 Connection to 2+1 field theory [Semenoff, DiVicenzo & Mele, Fradkin, Haldane, etc.]

~90’s Theory of carbon nanotubes [Dresselhauss, Dresselhauss, Saito, Ando, etc.]

~2000 Early work on graphene [Guinea et al., Kane and Mele, Ando, etc.]
III) Experimental techniques
III) Experimental techniques

Two techniques:

a) Mechanically exfoliated graphene

Recipe: Take a (monocrystallite) graphite sample (HOPG). Remove layers with a scotch tape and then press it on a Si/SiO\(_2\) substrate: you will deposit graphite flakes. A small fraction are monolayers. Detect them with an optical microscope (300nm SiO\(_2\) thickness): that’s tough. Contact the monolayer with metallic leads (gold e.g.). Apply an electric tension to the heavily n-doped Si backgate.

Novoselov et al., Science 2004 and PNAS 2005
III) Experimental techniques

**Electric field effect**: a gate tension $V_g$ allows to control the filling on electrons in the graphene sheet ("doping" of graphene)

Capacitor = plate(graphene)/dielectric(SiO$_2$)/plate(n-doped Si)

Novoselov et al., Science 2004 and PNAS 2005

Zhang et al., Nature 2005

![Graph showing electric field effect in graphene](image)

- Typical mobility $\mu \sim 1$ m$^2$/V.s
- Record mobility $\sim 20$ m$^2$/V.s in 2DEG $\sim 1000$ m$^2$/V.s

- Typical density $n \sim 10^{16}$ m$^{-2}$
- $n/V_g = 7 \times 10^{14}$ m$^{-2}$/V

**Graphical Data**

- $\mu (10^3 \text{cm}^2 \text{V}^{-1} \text{s}^{-1})$ vs $V_g (V)$
- $n_s (10^1 \text{cm}^{-2})$ vs $V_g (V)$
III) Experimental techniques

Suspended sheet and ripples

Typical height \( \leq 1 \text{ nm} \)
Typical length \( \sim \) tens of nm

*Meyer et al., Nature 2007*
b) Epitaxial graphene on SiC

Berger et al., J. Phys. Chem. 2004 [de Heer’s group]
IV) Atomic structure
IV) Atomic structure

Graphene = 2D honeycomb crystal of carbon.

Carbon atom: 6 electrons 1s² (core) 2s² 2p² (valence) 
hybridization: 1 × 2s orbital and 2 × 2p orbitals → 3 × sp² orbitals
1 × 2pₓ orbital left

- 3 coplanar σ bonds, with 120° angle: honeycomb structure
- 1 conduction electron per C atom, 2pₓ orbital, perpendicular to the plane, giving π bands: electronic properties
IV) Atomic structure

**Honeycomb crystal** = triangular (2D) Bravais lattice + 2 atoms basis (important for Bloch’s theorem)

**Direct space:**

- 2D triangular lattice: lattice vectors \((\tau_1, \tau_2)\)

  - lattice constant \(= \tau_1 = \tau_2 = a\sqrt{3} \approx 2.5 \text{ Å}\)
  - angle \(= (\tau_1, \tau_2) = 60^\circ\)

- Atomic basis: \(C_A(0, 0)\) et \(C_B(1/3, 1/3)\)

  \[ C - C \text{ distance} = a = 1.42 \text{ Å} \]

  - Nearest neighbors vectors \((e_1, e_2, e_3)\)

- Primitive unit cell contains 2 atoms, 1 \(C_A\) and 1 \(C_B\)

  \[ \Rightarrow 2 \text{ conduction electrons per primitive unit cell} \]

  number of primitive unit cells \(N_m = \mathcal{A}/(a^23\sqrt{3}/2)\)
IV) Atomic structure

Reciprocal space:
Reciprocal lattice of the triangular lattice = triangular lattice (whatever the atomic basis)

- Reciprocal lattice (RL) = 2D triangular lattice:
  lattice vectors \((g_1, g_2)\)
  lattice constant \(= g_1 = g_2 = \frac{4\pi}{3a}\)
  angle \(= (\hat{g_1}, \hat{g_2}) = 120^\circ\)

- primitive unit cell = 1st Brillouin zone
  center of 1Bz : \(\Gamma\)
  crystal momentum conservation \(\Rightarrow\)
  only two inequivalent corners of the 1Bz:
  we take \(K = \frac{4\pi}{3\sqrt{3}a} e_x = -K'\)
Conclusion:

- Direct space: sublattice index
  \[ l = A, B \]

- Reciprocal space: valley index
  (has no connection to the 2 carbon atoms in the basis, but is related to the Bravais lattice)
  \[ \xi = +1(K), -1(K') \]

- First 2D crystal: they do exist and they are stable (however they are not flat but rippled).
V) Band structure
Nearest neighbor tight binding model for the conduction electrons [Wallace 1947]

- nearest neighbor hopping amplitude: \( t \approx 3 \) eV
- no neighboring \( 2p_z \) orbital overlap
- \( 2p_z \) orbital energy: \( \varepsilon_{2p_z} = 0 \)
- 1 conduction electron per atom

\[ \Rightarrow \text{Hamiltonian (2nd quantization)}: \]

\[
\hat{H} = -t \sum_{R \in BL} \sum_{m=1}^{3} \hat{b}^\dagger_{R+e_m} \hat{a}_R + \text{h.c. where } R = \text{integer } \times \tau_1 + \text{integer } \times \tau_2
\]

to be diagonalized.

Bloch’s theorem (deals with the Bravais lattice (BL); not a FT) \( \Rightarrow \)

\[
\hat{a}^\dagger_R = \frac{1}{\sqrt{N_m}} \sum_{k \in 1Bz} e^{-i\mathbf{k} \cdot \mathbf{R}} \hat{a}^\dagger_{\mathbf{k}} \quad \text{et} \quad \hat{b}^\dagger_{R+e_m} = \frac{1}{\sqrt{N_m}} \sum_{\mathbf{k} \in 1Bz} e^{-i\mathbf{k} \cdot (\mathbf{R}+e_m)} \hat{b}^\dagger_{\mathbf{k}}
\]
V) Band structure

\[ \hat{H} = \sum_{\mathbf{k} \in 1Bz} h_{\mathbf{k}} \hat{b}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}} + \text{h.c. with } h_{\mathbf{k}} \equiv -t \sum_{m=1}^{3} e^{-i\mathbf{k} \cdot \mathbf{e}_m} = |h_{\mathbf{k}}| e^{i\theta_{\mathbf{k}}} \]

\[ = \sum_{\mathbf{k} \in 1Bz} \left( \hat{a}_{\mathbf{k}}^\dagger \hat{b}_{\mathbf{k}}^\dagger \right) \begin{pmatrix} 0 & h_{\mathbf{k}}^* \\ h_{\mathbf{k}} & 0 \end{pmatrix} \begin{pmatrix} \hat{a}_{\mathbf{k}} \\ \hat{b}_{\mathbf{k}} \end{pmatrix} \]

2\times2 matrix in sublattice (A,B) space

Then, a rotation in sublattice \((A, B)\) space:

\[ \hat{c}_{\mathbf{k},\alpha}^\dagger = \frac{1}{\sqrt{2}} \left( e^{-i\theta_{\mathbf{k}}/2} \hat{a}_{\mathbf{k}}^\dagger + \alpha e^{i\theta_{\mathbf{k}}/2} \hat{b}_{\mathbf{k}}^\dagger \right) \text{ where } \alpha = \pm 1 \]

so that the Hamiltonian becomes

\[ \hat{H} = \sum_{\mathbf{k} \in 1Bz} \sum_{\alpha = \pm 1} \varepsilon_{\mathbf{k},\alpha} \hat{c}_{\mathbf{k},\alpha}^\dagger \hat{c}_{\mathbf{k},\alpha} \text{ où } \varepsilon_{\mathbf{k},\alpha} \equiv \alpha |h_{\mathbf{k}}| \]

- the energy vanishes on two points, on the corners of the 1Bz:
  \[ \varepsilon_{\pm \mathbf{K},\alpha} \equiv \alpha |h_{\pm \mathbf{K}}| = 0 \]

- band index \( \alpha = +1(CB, \text{particle}); -1(VB, \text{hole}) \)

refers to the same space as the sublattice index \( l = A, B \)
**V) Band filling**

VB and CB meet on 2 points, corners of the 1Bz
(where there usually is a gap!)

Undoped graphene ($V_g = 0$) :
Number of single electron states per band = $2N_m$
Number of conduction electrons = $N_{el} = 2N_m$
$\Rightarrow \frac{N_{el}}{2N_m} = 1$ : VB is full and CB is empty.
Graphene is a gapless semiconductor!
Or a metal ($\varepsilon_F = 0$) with a vanishing density of states at the Fermi level!

The big band (CB+VB) is half-filled.

Doped graphene ($V_g \neq 0$) :
$N_{el} = 2N_m + N_c$ where $N_c = C_g V_g/e$
$\Rightarrow$ filling $= \frac{N_c}{2N_m} \propto V_g$
Metal with a small (tunable) density of states.

\[ \varepsilon_F \propto \sqrt{V_g} \text{ typically : } \varepsilon_F \sim 0.1t \text{ when } V_g = 100V \]

Orders of magnitude : typically $V_g \sim 1 \text{ V}$, $n_c \sim 10^{16} \text{ m}^{-2}$, $\varepsilon_F \sim 0.1 \text{ eV} \sim 1000 \text{ K}$
V) Electron and hole puddles

Close to $V_g = 0$, $N_c = 0$ globally but $n_c \neq 0$ locally because the sample is inhomogeneous. It is made of puddles of electrons and holes (one possible explanation: electrons locally screen charged impurities trapped in the substrate).

Consequences:
- residual density $n_c^{\text{min}} \sim 10^{14}$ to $10^{16}$ m$^{-2}$
- minimal Fermi energy $\varepsilon_F^{\text{min}} \sim$ hundreds of K

puddle size $\sim 30$ nm
$n_c^{\text{min}} \sim 2 \times 10^{15}$ m$^{-2}$
charge carrier per puddle $\sim 2$

**Figure 3** Spatial density fluctuations and electron/hole puddles. a, Colour map of the spatial density variations in the graphene flake extracted from surface potential measurements at high density and when the average carrier density is zero. The blue regions correspond to holes and the red regions to electrons. The

*Martin et al., Nature Phys. 2008*
VI) Low energy effective theory
VI) Low energy effective theory

Close to the $K$ point where $\varepsilon = 0 = \varepsilon_F$ (similarly close to the $K'$ point, except for a few signs), we expand the dispersion relation:

$$ k = K + p/\hbar \text{ with } p/\hbar \ll 1/a \text{ i.e. } |\varepsilon| \ll t. \text{ Hence }$$

$$ \varepsilon_{k,\alpha} \approx \alpha \frac{3}{2} t a |k - K| = \alpha v_F p $$

where $v_F \equiv 3ta/(2\hbar) \approx 10^6 \text{ m/s} \approx c/300$ is an “effective speed of light”.

The dispersion relation is linear (rather than parabolic) : therefore the effective electron mass $m^*$ vanishes! ($\varepsilon = \alpha \sqrt{m^* 2v_F^4 + p^2 v_F^2} = \alpha v_F p$)
Density of states (DoS) at the Fermi level, per unit area:

\[
\rho(\varepsilon_F) = \frac{2\varepsilon_F}{\pi \hbar^2 v_F^2} = \frac{2k_F}{\pi \hbar v_F} = \frac{2\sqrt{\pi|n_c|}}{\pi \hbar v_F} \propto \sqrt{V_g}
\]
VI) 2D massless Dirac Hamiltonian

The (2nd quantized) Hamiltonian becomes (with \( k = K + p/\hbar \)):

\[
\hat{H}_K \approx \sum_p (\hat{a}^\dagger_k \hat{b}^\dagger_k) \begin{pmatrix}
0 & v_F(p_x + ip_y) \\
v_F(p_x - ip_y) & 0
\end{pmatrix} \begin{pmatrix}
\hat{a}_k \\
\hat{b}_k
\end{pmatrix}
\]

\[
= \sum_p (\hat{a}^\dagger_k \hat{b}^\dagger_k)v_F p \cdot \sigma^* \begin{pmatrix}
\hat{a}_k \\
\hat{b}_k
\end{pmatrix}
\]

where \( \sigma_x \) and \( \sigma_y \) are the Pauli matrices.

\( \sigma = (\sigma_x, \sigma_y) \) is the sublattice spin.

Therefore the single electron Hamiltonian is a 2D massless Dirac Hamiltonian (with a \( 2 \times 2 \) matrix structure in sublattice space):

\[
H_K = v_F p \cdot \sigma^* = v_F \sigma^* \cdot (-i\hbar \nabla)
\]

Close to the \( K' \) point, one finds:

\[
H_{K'} = -v_F p \cdot \sigma
\]

Changing the representation: \( A \rightleftharpoons B \) and \( \psi = [KA, KB, K'B, K'A] \quad \text{quadri-spinor} \)

\[
H_\xi = \xi v_F p \cdot \sigma \quad [H = v_F p \cdot \sigma \otimes \tau^\text{valley}_z]
\]
The Dirac equation is

\[ i\hbar \partial_t \psi = H_D \psi \], with \( H_D = cp \cdot \alpha + mc^2 \beta \)

where \( \alpha \) has \( d \) components (\( d \)-space dimension): \( \alpha_1, \ldots, \alpha_d \). Let \( \alpha_{d+1} = \beta \).

In order to have the correct relativistic dispersion relation, Dirac requires that

\[ H_D^2 = c^2 p^2 + m^2 c^4 \]

which implies

\[ \{ \alpha_\mu, \alpha_\nu \} = 2\delta_{\mu,\nu} \text{ with } \mu = 1, \ldots, d + 1 \]

(Clifford algebra)
V) Dirac 3+1 versus Weyl 2+1

Usual Dirac equation 3+1: QED relativistic electron
\[ \varepsilon = \pm \sqrt{p^2 c^2 + m^2 c^4} \]
\[ H_D = cp \cdot \alpha + mc^2 / \beta \]
c = light velocity
\( \alpha_x, \alpha_y, \alpha_z, \beta \): four 4 \times 4 matrices
spin \( \uparrow / \downarrow \) and electron/positron

Massless Dirac (Weyl) 2+1: graphene massless electron (\( \sim \) charged neutrino)
\[ \varepsilon = \pm v_F p \]
\[ H_W = v_F p \cdot \alpha \ [\text{usually } \alpha = \sigma = (\sigma_x, \sigma_y)] \]
\( v_F = \) Fermi velocity \( \approx c / 300 \)
\( \alpha_x, \alpha_y, \beta \): three 2 \times 2 (Pauli) matrices
electron/hole (no spin)

Dirac equation 1928
Positron (Dirac 1930)
Fermi sea
Dirac sea

Weyl equation for neutrino 1929
Hole in semiconductors (Peierls 1929)
VI) Effective theory: summary

The $2 \times 2$ structure of the Dirac Hamiltonian is in sublattice space (sublattice index $l = A, B$), i.e. band space. Physically: one has to consider the two bands simultaneously and they are electron/hole symmetric.

There are 4 copies of the Dirac equation (4 types of electrons):

$4 = 2$(spin) $\times$ 2(valley)

$\Rightarrow 8 \times 8$ Hamiltonian $H = v_F \mathbf{p} \cdot \sigma \otimes \tau^\text{valley} \otimes \mathbb{I}^{\text{spin}}$.

Spin index $s_z = \uparrow, \downarrow$ and valley index $\xi = +1(K), -1(K')$.

In the absence of interactions, disorder, special boundary conditions, etc., the four types of electrons are (almost) independent and one only considers:

$$H = v_F \mathbf{p} \cdot \sigma \ (4 \text{ times})$$

Beware of “spin”, “pseudospin”, “isospin” that may refer to the (real) spin, or to the sublattice index, or to the valley index!
VII) “Relativistic” quantum mechanics in graphene
VII) Minimal localization length

Fundamental restriction for a “relativistic” quantum mechanical description of a single particle (Landau and Peierls 1931):
- quantum mechanics: $\Delta x \Delta p \geq \hbar$ and $v \equiv \partial E / \partial p = v_F$ imply $\Delta E \geq \hbar v_F / \Delta x$.
- “relativity”: two bands (particle and anti-particle) that touch (no gap)

A wavepacket that is too localized ($\Delta x \to 0$) has a large spread in energy ($\Delta E \to \infty$) and can no more be describing a single particle if $\Delta E$ becomes larger than $E$. Therefore, there is a minimum localization length:

$$\Delta x \geq \frac{\hbar v_F}{E} = \frac{1}{k} \sim \lambda$$

Similar to a photon: minimal resolution of an optical microscope is set by the wavelength. Experimental graphene: $\lambda_F \sim 5$ to 100 nm.

If one is interested in length smaller than the minimal localization length $(\Delta x)_{\text{min}} \sim \lambda$, one needs to consider a many particle theory: quantum field theory rather than relativistic quantum mechanics.
VII) Chirality (helicity) and sublattice spin

Chirality = helicity = projection of sublattice spin $\sigma$ on the direction of motion

$$c \equiv \sigma \cdot \frac{p}{p}$$

is the chirality operator. Its eigenvalues are $\gamma = \pm 1$. Chirality is conserved $[H_\xi, C] = 0$. The Hamiltonian $H_\xi = \xi v_F p \cdot \sigma = v_F p \xi C$. Eigenvectors of $H_\xi$ can be indexed as $|p, \gamma, \xi\rangle$:

$$\langle r, l, \xi'| p, \gamma, \xi\rangle = \delta_{\xi, \xi'} \frac{1}{\sqrt{A}} e^{i p \cdot r / h} \frac{1}{\sqrt{2}} \left\{ \begin{array}{ll} 1 & \text{if } l = A \\ \gamma e^{i \xi \theta_p} & \text{if } l = B \end{array} \right. \quad \text{and } \varepsilon_{p, \gamma, \xi} = \xi \gamma v_F p = \alpha v_F p$$

where $\tan \theta_p = p_y / p_x$ and $A$ is the sample area.

Therefore, band index $(\alpha) = \text{valley} (\xi) \times \text{chirality} (\gamma)$

Electrons in graphene are massless and chiral (meaning that the wavefunction has a spinorial structure).
VII) Velocity, current and Zitterbewegung

Velocity operator \( \hat{v} \sim \text{sublattice spin } \hat{\sigma} \)

\[
\hat{v} \equiv \frac{d\hat{\mathbf{r}}}{dt} = \frac{1}{i\hbar} [\hat{\mathbf{r}}, \hat{H}_\xi] = \xi v_F \hat{\sigma} \quad \text{(Breit 1928)}
\]

Therefore \( \hat{H}_\xi = \hat{v} \cdot \hat{p} \).

Velocity is not conserved \( [\hat{H}_\xi, \hat{v}] \neq 0 \Rightarrow \) Zitterbewegung (jittering motion).

Expectation value for a plane wave:

\[
\langle \hat{v} \rangle = \alpha v_F \frac{p}{p}
\]

Velocity is not proportional to momentum \( p \).

Probability density: \( \rho_\varphi(r) \equiv |\varphi(r)|^2 \).

Probability conservation \( \partial_t \rho_\varphi(r) + \nabla \cdot j_\varphi(r) = 0 \) implies that the mean current is:

\[
j_\varphi(r) = v_F \varphi^\dagger(r) \sigma \varphi(r)
\]
VII) Absence of backscattering

A smooth impurity potential at the lattice scale:

\[ H = \nu_F p \cdot \sigma \otimes \tau_z + V(r)I_{4\times4} \]

It can not scatter between valleys. It does not act on the sublattice spin \( \sigma \). It does act on \( p \).

Initial state \(|p,\gamma,\xi\rangle\). Final state \(|p',\gamma',\xi'\rangle\). Elastic scattering (\( p' = p \) and \( \alpha' = \alpha \)) and intravalley (\( \xi' = \xi \)). But \( \gamma = \xi \times \alpha \), therefore \( \gamma' = \gamma \). The only freedom is in the scattering angle \( \theta = (p',p) \).

Scattering probability at the Born approximation:

\[ P(\theta) \propto |\langle p',\gamma,\xi|V_{\text{imp}}|p,\gamma,\xi\rangle|^2 = |\tilde{V}_{\text{imp}}(q = p' - p)|^2 \cos^2(\theta/2) \]

Absence of backscattering: \( P(\pi) = 0 \).

VIII) Minimum about el-el interactions
VIII) Minimum about el-el interactions

1) **Non-retarded** \((c \gg v_F)\) 3D **Coulomb interactions**
But 2D Fourier transform \(\tilde{V}_0(q) = \frac{2\pi e^2}{\epsilon_r q}\)

2) **Interaction strength** \(r_s = \frac{\epsilon_{\text{int}}}{\epsilon_{\text{kin}}} = \frac{e^2}{\epsilon_r \hbar v_F} = \alpha_{\text{graphene}}\)
   - \(r_s \approx 1\) on SiO\(_2\) substrate
   - scale invariance: carrier density independent (2DEG: \(r_s = \frac{\epsilon_{\text{int}}}{\epsilon_{\text{kin}}} \sim \frac{me^2}{\epsilon_r \hbar^2 \sqrt{n_c}}\))
   - can be modified via \(\epsilon_r\) from 2.2 (in vacuum \(\epsilon_r = 1\)) to \(\approx 0\) when \(\epsilon_r \gg 1\)

3) **Screening by \(\pi\) electrons**
   - doped graphene: linearized Thomas-Fermi screening (as in 2DEG): \(\tilde{V}_{\text{TF}}(q) = \frac{2\pi e^2}{\epsilon_r (q + \kappa)}\). Screening length \(\sim 1/\kappa = 1/(4r_s k_F)\). Good screening if \(r_s > 1/4\). Bad screening if \(r_s < 1/4\).
   - undoped graphene: almost no screening (because TF screening length \(\to +\infty\)).

4) **Fermi liquid?**
   - doped graphene: yes with \(v^*_F \approx v_F [1 - \frac{r_s}{4} \log(k_F a)] > v_F\)
   - undoped graphene: no. Marginal Fermi liquid with \(v^*_F(k) \approx v_F [1 - \frac{r_s}{4} \log(ka)]\).
IX) Conclusions
IX) Solid state physics’ conclusion: a peculiar solid

- A 2D semiconductor but with zero gap: one cannot apply the standard theory of semiconductors ($\Delta=0$).

- A 2D metal but with zero density of states at the Fermi level (when undoped): one cannot apply the standard theory of metals ($\rho(\varepsilon_F)=0$).

- The electrons close to the Fermi level do not obey a Schrödinger equation with an effective mass. They obey a massless (2D) Dirac equation ($m^*=0$). One cannot apply the standard transport theory.
IX) "Relativistic" quantum mechanics' conclusion

- Graphene is a 2D gapless semiconductor with 2 valleys (K,K': fermion doubling).
- At low energy, its conduction electrons obey a 2D massless Dirac equation. Electron-hole symmetry + zero gap → chiral electrons.

References
- Popular scientific reviews: M. Wilson, Physics Today (January 2006), page 21; J.N. Fuchs, M.O. Goerbig, M. Potemski, Images de la physique (CNRS, 2007) et Pour la Science (mai 2008), page 36.