Disorder-induced transverse delocalization in ropes of carbon nanotubes

M. Ferrier, A. Chepelianskii, S. Guéron, and H. Bouchiat
Univ. Paris-Sud, CNRS, UMR 8502, F-91405 Orsay Cedex, France

(Received 11 February 2008; revised manuscript received 19 April 2008; published 14 May 2008)

A rope of carbon nanotubes consists in an array of parallel single wall nanotubes with nearly identical diameters. In most cases, the individual nanotubes within a rope have different helicities and 1/3 of them are metallic. In the absence of disorder within the tubes, the intertube electronic transfer is negligible because of the longitudinal wave vector mismatch between neighboring tubes of different helicities. The rope can then be considered as a number of parallel independent ballistic nanotubes. On the other hand, the presence of disorder within the tubes favors the intertube electronic transfer. This is first shown by using a very simple model wherein disorder is perturbatively treated as inspired by the work of Maarouf et al. [Phys. Rev. B 61, 11156 (2000)]. We then present numerical simulations of a tight binding model of a rope. A disorder induced transverse delocalization shows up as an increase (by typically 1 order of magnitude of the sensitivity to the transverse boundary conditions in the presence of small disorder). This is accompanied by an increase in the longitudinal localization length. The implications on the nature of electronic transport within a rope of carbon nanotubes are discussed.

DOI: 10.1103/PhysRevB.77.195420

PACS number(s): 73.63.Fg

I. INTRODUCTION

A rope of single wall carbon nanotubes (SWNTs) is generally made up of ordered parallel tubes with different helicities but with a narrow distribution of diameters.1,2 The center of the tubes form a triangular lattice so that for each metallic tube in a rope, there are, on average, two neighboring tubes that are also metallic. In the absence of disorder within the tubes, the intertube electronic transfer, which is defined as the matrix element of the transverse coupling between two neighboring tubes, integrated over spatial coordinates is negligible because of the longitudinal wave vector mismatch between tubes of different helicities.3 The rope can then be considered made up of parallel independent nanotubes. The transport is ballistic and the dimensionless conductance (in units of the quantum conductance $G_Q = 2e^2/h$) is two times the number of metallic tubes within the rope. However, it has been shown1,4 that the disorder within the tubes favors intertube scattering by relaxing the strict orthogonality between the longitudinal components of the wave functions. By using a very simple model wherein disorder is perturbatively treated, we show in Sec. II that the intertube scattering time is shorter than the elastic scattering time within a single tube. In tubes longer than the elastic mean free path, this intertube scattering can provide additional conducting paths to electrons, which would otherwise be localized in isolated tubes. In the limit of localized transport along the tubes, we show that the longitudinal localization length is not a monotonous function of disorder and it increases at moderate disorder. In order to go beyond these analytical results, we have performed numerical simulations on a tight binding model of coupled one-dimensional (1D) chains with different longitudinal hopping energies. This model described in Sec. III mimics the physics of transport in a rope of carbon nanotubes in the sense that in the absence of disorder, the electronic motion is localized within each chain. The transverse delocalization as a function of disorder is investigated through the sensitivity of eigenenergies to a change in transverse boundary conditions from periodic to antiperiodic.

These results show that disordered ropes of carbon nanotubes can be considered as anisotropic diffusive conductors, which, in contrast to individual tubes, exhibit a localization length that can be much greater than the elastic mean free path.

II. ELECTRONIC STRUCTURE OF ROPES OF CARBON NANOTUBES

A. Band structure of a rope without disorder

We consider a rope constituted from SWNTs with diameters ranging between 1.2 and 1.5 nm. It can be shown (see Table I) that the tubes within such a rope can have different kind of helicities. By following the model developed by Maarouf and Kane,3 one can characterize the electron wave functions at the Fermi energy $\epsilon_F$ with two wave vectors $k_\perp$

<table>
<thead>
<tr>
<th>Helicity $(n, m)$</th>
<th>Diameter (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(9,9)</td>
<td>1.24</td>
</tr>
<tr>
<td>(10,10)</td>
<td>1.38</td>
</tr>
<tr>
<td>(11,8)</td>
<td>1.32</td>
</tr>
<tr>
<td>(12,9)</td>
<td>1.45</td>
</tr>
<tr>
<td>(12,6)</td>
<td>1.26</td>
</tr>
<tr>
<td>(13,7)</td>
<td>1.4</td>
</tr>
<tr>
<td>(15,6)</td>
<td>1.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Helicity $(n, m)$</th>
<th>Diameter (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(14,5)</td>
<td>1.36</td>
</tr>
<tr>
<td>(13,4)</td>
<td>1.23</td>
</tr>
<tr>
<td>(16,4)</td>
<td>1.46</td>
</tr>
<tr>
<td>(15,3)</td>
<td>1.33</td>
</tr>
<tr>
<td>(17,2)</td>
<td>1.44</td>
</tr>
<tr>
<td>(16,1)</td>
<td>1.32</td>
</tr>
<tr>
<td>(18,0)</td>
<td>1.43</td>
</tr>
</tbody>
</table>

TABLE I. Values of possible helicities and diameters of the 14 metallic tubes with a diameter between 1.5 and 1.2 nm. The diameter and helicity are related through $D(n,m)=\frac{0.25}{\pi}\sqrt{m^2+mn+n^2}$. There are also 24 insulating tubes in the same diameter range.
and \( k_1 \) that are perpendicular and parallel, respectively, to the tube axis; \(|\Psi\rangle = |k_\| \rangle |k_\rangle\), such that \(\Psi(x,y) \sim e^{ik_\|x} e^{ik_\|y}\). From this model, it is possible to compute the matrix elements of the transverse coupling Hamiltonian \( H_{\perp} \) between two nearest-neighbor tubes \( a \) and \( b \),

\[
\langle \Psi_{a} | H_{\perp} | \Psi_{b} \rangle = t_{\perp} (a,b) \delta_{k_{a}a_{\perp}k_{b}a_{\perp}},
\]

(1)

where \( t_{\perp} (a,b) = t_{F} \gamma (1/4) R_{a_{\perp}k_{a_{\perp}a_{\perp}}^2} \) and \( t_{F} = t_G \sqrt{a_0/\pi \epsilon} \) is the Fermi energy illustrating the orthogonality between wave functions of different helicities, the electronic transport is confined within each tube axis; \( E = 7.5 \text{ meV} \) for an average tube radius \( R = 0.7 \text{ nm} \), \( t_{G} = 0.1 \text{ eV} \) is the interplane hopping energy in graphite, and \( a_0 = 0.5 \times 10^{-10} \text{ m} \) is the Bohr radius. The term \( \delta_{k_{a}a_{\perp}k_{b}a_{\perp}} \) is due to the orthogonality between wave functions of different longitudinal wave vectors and shows that the tubes of different helicities are uncoupled to first order in \( H_{\perp} \). The exponential part takes into account that the \( \sigma \) orbitals belonging to the neighboring tubes of different helicities.\(^3\)

By going beyond first order perturbation, it is possible to show that the neighboring tubes of different helicities are coupled to second order in \( H_{\perp} \) with a characteristic energy scale \( e_{\perp,2} = 2 t_{\perp}(a,b)^2/\delta \epsilon \) (Fig. 1) involving transitions to higher energy states for which \( \epsilon_{a}(k) = \epsilon_{b}(k) = \epsilon_{F} + \delta \epsilon(a,b)/2 \). This second order coupling gives rise to an intertube hopping probability that is very small compared to the inverse ballistic time \( L/v_F \) of a micron long nanotube. Thus, it is reasonable to assume that in a rope constituted of tubes of different helicities, the electronic transport is confined within each ballistic tube and exhibits a strong 1D character.

\[\text{B. Disordered ropes in the perturbative regime}\]

In the presence of disorder, plane waves that are localized on individual tubes are perturbed into wave packets and become much more sensitive to the transverse coupling leading to a transport regime that is no longer 1D but can be delocalized on percolating clusters of metallic tubes within the rope.

We consider a very short range (on site) disorder potential, such that its average \( \langle V(x) \rangle = 0 \) and its variance \( \langle (V(x))^2 \rangle = W \).

\[
V(x) = \sum_{a,x_{a}} W(x_{a}) \delta(x - x_{a}),
\]

(2)

where the index \( a \) runs on the chains constituting the rope, each of them characterized by its atomic sites \( x_{a} \) and the disorder potential \( W(x_{a}) \), the probability distribution of which is given by

\[
P(W(x_{a})) = \begin{cases} 
1/W, & \text{if } w(x_{a}) \in [ -W/2; W/2 ]; \\
0 & \text{otherwise.}
\end{cases}
\]

The disorder perturbed wave functions can be written to first order in disorder as

\[
|\Psi'\rangle = |k_{\underline{a}}\rangle \left[ |k_{\underline{b}}\rangle + \sum_{i} \frac{\tilde{V}(k_{\underline{a}} - k_{\underline{i}})}{\epsilon_{i} - \epsilon_{\underline{a}}} |k_{\underline{i}}\rangle \right],
\]

(4)

where \( i \) runs over the unoccupied states and \( \tilde{V}(k) \) is the Fourier component of \( V \) at the wave vector \( k \). We do not consider transitions involving different values of \( k_\| \) and neglect possible perturbation of \( t_{\perp} (a,b) \) with disorder.

This perturbed wave function contains plane waves of all values of parallel momentum. As a result, electrons can hop from tube to tube and conserve their momentum. The intertube coupling energy between two tubes \( a \) and \( b \) is now \( E_{\perp} = \langle \Psi_{\underline{a}}' | H_{\perp} | \Psi_{\underline{b}}' \rangle \), which reads to first order in disorder potential as

\[
E_{\perp} (a,b) = \langle k_{\underline{a}} | H_{\perp} | k_{\underline{b}} \rangle \frac{2}{L} \sum_{x_{a}} W(x_{a}) \cos \left[ (2 \pi \lambda_{\perp} - k_{\underline{a}} x_{a}) \right] \frac{\epsilon_{\underline{i}}(k_{\underline{i}}) - \epsilon_{\underline{a}}(k_{\underline{a}})}{\epsilon_{i} - \epsilon_{\underline{a}}},
\]

(5)

The disorder average value of this coupling is zero but its typical value \( e_{\perp} = \sqrt{e_{\perp,2}} \) is equal to

\[
e_{\perp} (a,b) = t_{\perp} (a,b) \frac{W}{3 \delta \epsilon(a,b)},
\]

(6)

where \( \delta \epsilon(a,b) = \hbar v_F |k_{\underline{i}} - k_{\underline{a}}| \).

This disorder induced intertube coupling energy is related to the second order coupling term calculated in Sec. II A through \( e_{\perp,1} (a,b) / e_{\perp,2} (a,b) = W / t_{\perp} (a,b) \), which, as will be shown below, can be much larger than 1 in a typical rope of SWNT. The coefficients \( t_{\perp} (a,b) \) and \( t_{\perp} (a,b) / \delta \epsilon(a,b) \) were calculated for all of the helicities corresponding to the metallic tubes given in Table I with a diameter between 1.2 and 1.5 nm and are depicted in the histogram in Fig. 2. From these values, we obtain the average value \( e_{\perp} = 0.03 W \).

To investigate the nature of transport in ropes, it is interesting to compare the typical intertube hopping time \( \tau_{\| h} = \hbar / (e_{\perp} \lambda_{\perp}) \) to the intratube scattering time \( \tau_{\| s} \) that is induced by the same disorder. The related elastic mean free path \( l_{e} = v_{F} \tau_{s} \) was calculated by White and Todorov\(^3\) and found to be given by \( l_{e} = \frac{e_{\perp}}{W} n_{C} \) for a tube of \( n_{C} \) carbon atoms along the circumference, where \( e_{\perp} = 2.7 \text{ eV} \) is the Fermi energy (measured from the bottom of the band) and \( W^2 = \text{the variance of} \)
DISORDER-INDUCED TRANSVERSE DELOCALIZATION IN

the disorder (assumed to be short range). This value of \( l_e \) is unusually large compared to what is expected in an ordinary conductor, since it is proportional to the number of sites along the circumference of the tube. This is due to the existence of only two conduction modes at the Fermi energy regardless of the diameter of the tube. As a result, there exists a rather large range of disorder \( W \) for which \( \tau_e \) is greater than \( \tau_b = h/(\langle e \rangle) \), which means that a charge carrier can visit several neighboring tubes between two elastic collisions. As schematically shown in Fig. 3, which compares the relevant time scales for transport in a rope of micron length, four different regimes (1, 2, 3, and 4) can be reached as the amplitude of disorder is increased.

1. At a very low amplitude of disorder, when both \( \tau_e \) and \( \tau_b \) are long compared to the ballistic time \( \tau_b = L/v_F \), the transport is ballistic and one-dimensional. Electronic wave functions are localized within single tubes.

2. When \( \tau_b < \tau_e < \tau_b \), the transport is still ballistic but the wave functions are delocalized over several tubes.

3. When \( \tau_b < \tau_b < \tau_e \), the transport is diffusive along the percolating clusters of metallic tubes along the rope as long as the rope is shorter than the localization length, the typical value of which is \( \xi = 2N_m \rho \), where \( N_m \) is the number of metallic tubes in the largest percolating cluster of metallic tubes within the rope. Typical values of \( N_m \) for commonly investigated SWNT ropes in experiments are given in the Appendix.

4. At large disorder \( \tau_b < \tau_e < \tau_e \), electronic states are localized within individual tubes at the scale of \( l_e \) (not shown in Fig. 3).

From this qualitative model, one expects that the transverse transport, but possibly also the longitudinal transport, is favored when increasing the disorder in the rope. In Sec. III, we present the numerical simulations that confirm this statement.

C. Analytical results in the localized regime

In the following, we consider the case wherein in the absence of interchain coupling, electronic wave functions are localized within each tube that is aligned along the \( x \) axis and can be characterized by the set of parameters \( x_a \), \( k_a \), and the localization length \( \xi_a \), such that a typical wave function on tube \( a \) reads,

\[
\Psi_a(x) = \cos(k_a(x-x_a)) \exp -\left(\frac{|x-x_a|}{\sqrt{\xi_a}}\right) \ ,
\]

which can also be written by the Fourier transform,

\[
\Psi_a(x) = \sum_k \exp ik(x-x_a) \{ f_a(k-k_a) + f_a(k+k_a) \},
\]

where \( f_a \) is a Lorentzian function centered on \( k_a \) of width \( \delta k_a = 2\pi/\xi_a \). In the presence of a small intertube coupling such as described by Eq. (1), one can easily compute the typical transverse coupling energy between two nearest neighbor tubes at the lowest order in \( t_{\perp} \),

\[
\langle \Psi_a^{\dagger} H_{\perp} \Psi_b \rangle = \frac{4t_{\perp} \exp ik(x_a-x_b)\lambda^4}{\lambda L} \sum_k \frac{\exp ik(x_a-x_b)\lambda}{(\lambda^2 + (k \pm k_b)^2)^2}.
\]

In the following, we assume that \( \delta k_a = \delta k_b = \lambda = 2\pi/\xi \) and the summation \( \Sigma_k \) runs on multiple values of \( 2\pi/L \) up to \( n = L/a_0 \), the number of sites on the tubes of length \( L \). Taking the square of Eq. (8) leads to a double summation on two wave vectors \( k \) and \( k' \) in which only diagonal terms contribute after averaging over the random phase factors \( \exp ik(k') \).

FIG. 3. Different transport regimes depending on the amplitude of disorder compared to a rope of length \( L = 1\mu_m \) as discussed in the text. Region 1—1D ballistic; region 2—3D ballistic; and region 3—3D diffusive. The arrows indicate the direction of increasing time. In region 2, the transport corresponds to a 3D quasi-ballistic regime on the whole length of the tube. On the other hand, in region 3, the transport is 3D ballistic at short time scales but becomes diffusive before reaching the end of the tube.
to be small compared to the Fermi energy $E_F$ terms centered around coupled, tight binding chains of different longitudinal energies $t_1 \neq t_2 \neq t_3 \cdots$. Right: The chains lie on a cylinder threaded by a fictitious Aharonov–Bohm flux, which enables the investigation of transverse transport.

$-k'(x_n - x_{n'})$ for $k \neq k'$. This yields the average square of the transverse coupling

$$\Gamma_{ab} = |\langle \Psi_a | H_\perp | \Psi_b \rangle|^2 = 16t_\perp (a, b)^2/(\lambda L)^2 \sum_k \frac{\lambda^8}{\lambda^2 + (k \pm k_a)^2} \frac{\lambda^2 + (k \pm k_b)^2}{\lambda^2 + (k \pm k_b)^2}.$$  

By keeping in the summation over $k$, only the $\lambda/(2\pi/L)$ terms centered around $k_a$ and $k_b$ with $\delta k = \lambda$, finally yields

$$\Gamma_{ab} = 4[t_\perp (a, b)^2/\lambda L^2] \frac{\lambda^2}{\pi L^2 [\lambda^2 + (k_a - k_b)^2]}.$$  

$\lambda$, the inverse typical localization length for 1D disordered chains, can be approximated by $\lambda = 4\alpha_{01}^{-1} W^2/E_F^2$, where the amplitude $W$ of the intrachain on site disorder is assumed to be small compared to the Fermi energy $E_F$. One can see from expression (10) that the typical transverse coupling between tubes of different helicities $k_a \neq k_b$ obtained after averaging on the positions $x_n$ and $x_{n'}$ increases with disorder like $W^2$ at low disorder and decreases like $1/W^2$ at large disorder such that $\lambda \gg |k_a - k_b|$.

III. SIMPLIFIED ANISOTROPIC TIGHT BINDING MODEL FOR A ROPE OF SINGLE WALL CARBON NANOTUBES

In the following, we investigate a simplified tight binding model for the transport in a rope containing a percolating cluster of $N_m$ metallic SWNTs. As shown in the Appendix, we expect that in the commonly investigated SWNT ropes, $N_m$ is at most equal to 10. Each SWNT labeled $n$ is described by a simple 1D atomic chain of $N_s$ sites with a nearest-neighbor coupling energy along the chain $t_n$, which can be different from one chain to the other. The variables $t_n$ are randomly distributed around their average value $t_\perp$ with a square distribution of width $\delta t_n$. For convenience, we take the chains on the surface of a cylinder where only sites belonging to nearest-neighbor chains are coupled (see Fig. 4). This crudely reproduces the situation of a hexagonally packed rope, wherein each metallic tube has an average of two metallic tubes as nearest neighbors. The interchain transverse coupling is described by a transverse nearest-neighbor coupling $t_\perp \ll t_\parallel$. The distribution among the $t_n$ plays the same role as the helicity distribution among the tubes in a rope, i.e., the interchain coupling becomes zero to the first order in $t_\perp$ since identical longitudinal Bloch wave vectors correspond to different energies $t_c \cos k_0 a_c$ except at half filling $k_0 a_c = \pi/2$. Note, however, that this disorder among the $t_n$ also implies a distribution of Fermi velocities that does not exist in carbon nanotubes. This leads to the following Hamiltonian: $H = H_t + H_\perp$, where

$$H_t = \sum_{n=1}^{N_m-1} \sum_{n'=1}^{N_s} t_{n,n'} |n,n_t\rangle \langle n,n_{t+1}| + H.c.,$$  

$$H_\perp = t_\perp \sum_{n=1}^{N_s} \sum_{n'=1}^{N_s} |n,n_t\rangle \langle n+1,n_{t+1}| + \exp(i\phi) |n_m,n_s\rangle \times \langle 1,n_{t+1}| + H.c.$$  

The last term in $H_\perp$ corresponds to the periodic boundary conditions around the cylinder modeling the rope. This periodic boundary condition involves a phase factor $\exp(i\phi)$ equivalent to a fictitious flux $\Phi$ through the cylinder that modifies the phase $\phi = 2\pi\Phi/\Phi_0$ of the transverse boundary conditions of the wave functions. Due to the different values of $t_n$ the wave functions are localized within each chain in the limit of very long chains $N_t \gg N_s$ and $t_\perp \ll t_\parallel$. An extra disorder Hamiltonian $H_d$ is added either as a random distribution of on site potentials $\psi_t$ of width $W$ or as an extra random contribution $\delta n_t(s,s+1)$ to the nearest-neighbor coupling $t_n$ within the chain $n$ (bond disorder) characterized by a distribution of width $\delta n_t$ assumed to be independent of $n$. Note, some analogy of this model with tight binding Hamiltonian with an anisotropic Anderson type of disorder investigated by Ping Sheng et al. However, in contrast to the situation of ropes of carbon nanotubes, the model used in that work is completely isotropic in the absence of disorder. Moreover, the authors consider a longitudinal Anderson type of disorder instead of the disorder on longitudinal hopping integrals that we have considered, which is closer to the physical situation of carbon nanotubes.

A. Numerical results

In order to investigate the interchain localization and the effect of disorder along the chains, we have calculated the eigenvalues of $H(\phi) = H_t(\phi) + H_d$. As shown in the seminal work of Thouless, the sensitivity of these eigenvalues to a change in the phase $\phi$ of the boundary conditions can be considered as a measure of the delocalization of the wave functions on the various chains constituting the rope. More precisely, we have computed the quantity $\delta n_t = \langle |\epsilon_n(\phi = \pi) - \epsilon_n(0)| \rangle$, where the average $\langle \cdot \rangle$ is taken on the $N_m N_s/4$ energy levels of the spectrum between 1/8 and 7/8 filling excluding the region between 3/8 and 5/8 filling. Around half filling, the tight binding dispersion relations for all chains are indeed crossing each other whatever the values of $t_n$ are and the model is inadequate. For ropes of $N_m = 10$ chains of $N_s$
DISORDER-INDUCED TRANSVERSE DELOCALIZATION IN... PHYSICAL REVIEW B 77, 195420 (2008)

FIG. 5. Transverse delocalization by disorder: evolution of \( \delta \varepsilon_{\text{perp}} \) with the amplitude of disorder for 10 coupled chains of 100 sites, \( W \) corresponds either to on site disorder (black dots) or to bond disorder (open circles). The situation of identical values of \( t_n \) corresponding to \( \delta t = 0 \) (upper curves) exhibits expected disorder induced localization, whereas the situation with different values of \( t_n \) where \( \delta t / t = 0.2 \) shows a regime of disorder induced transverse delocalization. The transverse hopping energy is chosen to be \( t_{\perp} = 0.06 t_c \). \( v \) is the density of states (inverse nearest level spacing).

FIG. 6. (Color online) Results obtained by diagonalization of a system of \( N_m = 10 \) chains of \( N_s = 1,013 \) sites. Continuous curves: Average energy difference \( \delta \varepsilon_{\text{perp}} \) as a function of on site disorder amplitude. From top to bottom: \( t_{\perp} / t_{\parallel} = 0.1, 0.07, 0.03, 0.02, \) and 0.01. Note the dependence in \( t_{\perp} ^{10} \) at low disorder. Dashed curve: Power law fits approximation of those curves with 2.5 and \(-9\) exponents (left and right parts).

FIG. 7. (Color online) Results obtained by diagonalization of a system of \( N_m = 10 \) chains of \( N_s = 1,013 \) sites. Average typical energy difference \( \delta \varepsilon_{\text{perp}} \) between the symmetric and/or antisymmetric boundary conditions in the longitudinal direction as a function of \( t_{\parallel} / t_{\parallel} \). From top to bottom \( W / t_{\parallel} = 0.55, 0.9, 1, 1.1, 1.3, \) and 1.5.

shown in Fig. 6, the conditions of observation of this nonmonotonic dependence of \( \delta \varepsilon_{\text{perp}} \) as a function of disorder amplitude drastically depends on the amplitude of the transverse hopping integral. It is clearly observed for very small values of \( t_{\perp} \) with low disorder increase in \( W^{2.5 \pm 0.5} \) followed by a decrease in \( 1 / W^{0.2 \pm 1} \). This power-law exponent is consistent with the analytical result derived in Sec. II C for the quantity \( \Gamma^{1/2} \times 1 / W \) describing the large disorder hopping between two adjacent tubes, its extension to hopping processes around a rope containing \( N_m \) chains yielding a decrease in \( 1 / W^{9/n} \). When increasing \( t_{\perp} \), multiple order hopping processes in \( t_{\perp}^{10} \) dominate the transverse transport and \( \delta \varepsilon_{\text{perp}} \) becomes independent of disorder at a low value of \( W \).

The sensitivity to a phase shift along the chain direction was also investigated from the computation \( \delta \varepsilon_{\text{par}} = |< \psi_\parallel (\phi_\parallel = \pi) - \psi_\parallel (0) | >_k \), where \( \phi_\parallel \) is the phase factor on the periodic boundary conditions parallel to the tube axis. For long ropes (1000 sites along the longitudinal axis), it is possible to observe an increase in \( \delta \varepsilon_{\text{par}} \) with \( t_{\perp} \) (see Fig. 7). This behavior is associated with an increase in the longitudinal localization length with \( t_{\perp} \) in the range of disorder where the intertube coupling increases with disorder. One can easily deduce from Fig. 7 that for the value of \( W = 1.3 \), the localization length \( \xi(t_{\perp}) \) increases by approximately a factor of 4 at \( t_{\perp} / t_{\parallel} = 0.06 \). This is done by assuming an \( \exp[-L / \xi(t_{\perp})] \) behavior for \( \delta \varepsilon_{\text{par}} \). In any case, \( \delta \varepsilon_{\text{par}} \) (contrary to \( \delta \varepsilon_{\text{perp}} \)) remains a monotonous decaying function of disorder. The quantity \( \delta \varepsilon_{\text{perp}} \) is very small in the absence of disorder, which is not the case for \( \delta \varepsilon_{\text{par}} \). This is due to the existence of the disorder among the \( t_n \) and also to the strong anisotropy between the longitudinal and transverse integrals, which makes the second order transverse hopping processes very small compared to first order ones. The nonmonotonic behavior \( \delta \varepsilon_{\text{perp}} \) is indeed observed only within the lowest order of perturbation in \( H_{\parallel} \) and tends to disappear when the higher order processes become important at higher values of \( t_{\perp} \) (see Fig. 6).

IV. CONCLUSION: IMPLICATION FOR THE TRANSPORT IN ROPES ON CARBON NANOTUBES

We have shown that a rope of single wall carbon nanotubes of different helicities is expected to exhibit very differ-
ent regimes of electronic transport depending on the amount of disorder. When the disorder is very small, the electronic transport takes place in independent 1D ballistic tubes. The conductance is then expected to be $G = 2NG_0$, where $N$ is the number of connected metallic tubes. The experimental observation of a strong shot noise reduction in short low resistive ropes confirms the ballistic nature of transport in these ropes. On the other hand, disordered tubes are expected to behave as three-dimensional (3D) diffusive multichannel conductors whose maximum value of conductance is $G = 2N_m(l/L)G_0$ when $L$ is smaller than the localization length $\xi = 2N_m l$, where $N_m$ is the number of metallic tubes in the largest percolating cluster of metallic tubes in the rope. Four probe transport measurements on $NT$ ropes after ion irradiation damage have been shown to give information on the intertube hopping processes within the rope which was found to increase with disorder. These findings can also be explained with the increase on the intertube scattering rate with disorder. The physics of intertube transfer has also been shown to play an important role in the transport in multiwall nanotubes.\textsuperscript{12,13} The situation is, however, different since each tube is only coupled within the first order to two other tubes (nearest inner and outer shells), and moreover, only the most external tube is connected to electrodes.

Let us also mention that a similar scenario of disorder induced delocalization has been predicted to take place in networks of disordered polymers, as discussed in Refs. 14–16.

We finally note that these different types of transport are also expected to influence the superconductivity as observed at a very low temperature on ropes of carbon nanotubes. The superconductivity in weakly disordered ropes has been observed to exhibit a strongly 1D character with a $T=0$ and $H=0$ transition. On the other hand, more resistive ropes exhibit a broad transition at finite temperature characteristic of a multichannel quasi-3D system.\textsuperscript{17,18}

ACKNOWLEDGMENTS

We acknowledge the very fruitful discussions with Chris-


FIG. 8. Size distribution of the number of tubes $n$ in a percolating cluster of metallic tubes for ropes containing $N=100$ and $N=400$ tubes among which $1/3$ on average are metallic.

APPENDIX: DETERMINATION OF THE TYPICAL SIZE OF PERCOLATING CLUSTER OF METALLIC TUBES WITHIN A ROPE

Carbon nanotubes in a rope are arranged according to a triangular network, with an average of $1/3$ of metallic tubes and $2/3$ of semiconducting ones. Since $1/3$ is below the percolating threshold, $1/2$ for nearest-neighbor couplings in the two-dimensional triangular lattice, the metallic tubes do not percolate over the full rope and constitute disconnected clusters whose size distribution depends on the number of tubes within the rope. We have numerically calculated this size distribution for ropes containing a few hundred tubes. This result is shown in Fig. 8 for ropes containing 100 and 400 tubes. The size distribution exponentially decays with the number of tubes in a cluster. We find that for a rope containing 400 tubes, the average cluster size is three but by integrating the number of clusters of size above a given value we find that there is at least one cluster of tubes of size above 15.