Superfluid to Mott-insulator of cold atoms in optical lattice

N. Dupuis - LPTMC Paris VI

Collaboration: K. Sengupta - Saha Institute (India)

Discussion: F. Gerbier - ENS (Paris).
Ultracold quantum gases

300 K
1 K
1 mK
1 µK
1 nK

300 K
1 K
1 mK
1 µK
1 nK

Laser cooling
Evaporative cooling in a magnetic trap

\[ ^{87}\text{Rb}\,\, T_{\text{BEC}} \sim 150 \, \text{nK} \]
\[ N \sim 10^6 \, \text{atoms} \]
\[ \frac{E_{\text{int}}}{E_{\text{kin}}} \sim n^{1/3} a \ll 1 \]
\( (a : \text{s-wave scattering length}) \)

Weakly interacting quantum gases
Optical lattices and strong correlations

3 pairs of counter-propagating laser beams

$$V_{OL}(r) = -\frac{1}{2} \alpha(\omega) E^2(r,t)$$

$$= V_0 [\sin^2(kx) + \sin^2(ky) + \sin^2(kz)], \quad k = 2\pi / \lambda$$

Analogous to the crystalline lattice in solids (but with no phonons!)

Lattice spacing: \( \frac{\lambda}{2} \sim 425 \ \text{nm} \)

Magnetic trap + optical lattice
Bose-Hubbard model

\[ H = \int d\mathbf{r} \psi^+(\mathbf{r}) \left( -\frac{\hbar^2}{2m} \nabla^2 + V_{\text{ext}}(\mathbf{r}) \right) \psi(\mathbf{r}) + \frac{4\pi \hbar a}{m} \int d\mathbf{r} \psi^+(\mathbf{r}) \psi^+(\mathbf{r}) \psi(\mathbf{r}) \psi(\mathbf{r}) \]

Expand over Wannier functions (lowest Bloch band):

\[ \psi(\mathbf{r}) = \sum_i w(\mathbf{r} - \mathbf{r}_i) \psi_i \]

\[ H = -t \sum_{\langle i, j \rangle} (\psi_i^+ \psi_j + \text{c.c.}) + \frac{U}{2} \sum_i \psi_i^+ \psi_i^+ \psi_i \psi_i \]

\[ t = \frac{4 E_r}{\sqrt{\pi}} \left( \frac{V_0}{E_r} \right)^{3/4} e^{-2 \sqrt{V_0/E_r}}, \quad U = \sqrt{\frac{8}{\pi}} k a E_r \left( \frac{V_0}{E_r} \right)^{3/4} \quad (E_r = \frac{\hbar^2 k^2}{2m}, \quad k = \frac{2\pi}{\lambda}) \]

No adjustable parameter.

The ratio \( t/U \) can be changed by varying the depth \( V_0 \) of the optical lattice (i.e. the laser light intensity).
Phase diagram of the Bose-Hubbard model (no magnetic trap)

Competition between tunneling and on-site interaction $U$

$t \gg U$

Superfluid ground state

$\Delta n_i \sim \langle n_i \rangle$

Macroscopic wave function $\Psi(r)$ with phase coherence

$t \ll U$

and integer filling (e.g. $n=1$)

Mott insulator ground state

$\langle n_i \rangle = 1$

$\Delta n_i \sim 0$
Phase diagram of the Bose-Hubbard model:
MI for integer fillings.

Fisher et al., PRB '89
Imaging ultracold quantum gases

Free fall:
$\approx$ a few $\text{ms}$
or a few $\text{10 ms}$

$\text{CCD Camera}$

Figure courtesy of F. Gerbier

\[
\begin{align*}
  n(r, t) & \sim |\tilde{\psi}(k)|^2 n_k \quad \text{with} \quad k = \frac{mr}{t} \\
  n_k &= \sum_{i, j} e^{ik \cdot (r_i - r_j)} \langle \psi_i^+ \psi_j \rangle \quad \text{momentum distribution}
\end{align*}
\]

MI: $n_k \sim \text{const}$

SF: periodic array of coherent matter-wave sources
$\Rightarrow$ sharp peaks at the reciprocal lattice vectors.
(analogous to Bragg peaks in the static structure factor of a solid).
Experimental observation of the SF-MI transition

3D gas: $^{87}\text{Rb}$


[Stoferle et al, Gerbier et al, Spielman et al., etc.]
Momentum distribution in a 2D Mott insulator


(no adjustable parameter!)

FIG. 3 (color online). Cross sections of normalized quasimomentum distributions (along $\hat{x} + \hat{y}$, and offset for clarity) at three values of $t/U$. The data are plotted along with the theoretical profile (solid lines) [28]. The dashed lines (not visible in the bottom trace) reflect the uncertainty in the theory resulting from the single-shot $\pm 0.5E_R$ uncertainty in the lattice depth.
Role of the magnetic trap (Jaksch et al. 1998)

Smoothly varying chemical potential: \[ \mu_{\text{loc}}(r) = \mu - V_{\text{ext}}(r) \]

Incompressibility!

SF with non-uniform density

MI: shell structure due to incompressibility \( (\partial n / \partial \mu = 0) \)

(Figures: courtesy of F. Gerbier)
Experimental observation of the incompressibility

Spatially selective microwave transitions
S. Folling, A. Widera, T. Muller, F. Gerbier, and I. Bloch (2006)

Theoretical integrated density profile

![Graphical representation of density profiles and labels](image)

- Total density
- n=1 sites
- n=2 sites
Excitation spectrum...

- SF phase: sound mode (observed in the absence of the optical lattice by two-photon Bragg spectroscopy).
- Mott insulator: gapped mode. No convincing observation so far.

... not observed yet.
RPA theory of the Bose-Hubbard model

[Shesdadri et al., Sengupta et al., Ohashi et al., Konabe et al, Menotti et al., etc.]

Mean-field decoupling of the hopping term

\[ \psi_i^+ \psi_j \rightarrow \phi_i^* \psi_j + \psi_i^+ \phi_j - \phi_i^* \phi_j \quad \text{with} \quad \phi_i = \langle \psi_i \rangle \quad \text{order parameter} \]

RPA: Gaussian fluctuation of \( \phi_i \)

\[ G^{-1}(q, \omega) = G^{-1}_{\text{loc}}(\omega) + t(q) \]

Exact for \( t=0 \), analogous to strong-coupling \( t/U \) expansion

Valid both in SF and MI phases

Phase diagram, momentum distribution, excitation spectrum
Excitation spectrum

MI: gapped mode

SF: sound mode + gapped mode (different from Bogoliubov theory)

Sengupta & ND 2005, Konabe et al. 2006
Ohashi et al. 2006, Huber et al. 2007,
Menotti & Trivedi 2008

Menotti & Trivedi 2008
Critical behavior at the SF-MI transition

Effective action near the transition

\[ S = \int_0^\beta d\tau \int d^d r \left[ \frac{Z}{\partial_x} \nabla \psi \right]^2 + \frac{Z_1}{\partial_x} \psi^* \partial_x \psi 
+ V \left| \partial_x \psi \right|^2 + r_0 \left| \psi \right|^2 + u \left| \psi \right|^4 \]
RG studies of the superfluid phase (no optical lattice)

- Bogoliubov theory plagued with IR divergences (Gavoret-Nozières 1964).
- Bogoliubov theory not correct in the infrared for $d<=3$ (Nepomnyashchii 1975)
- Non-perturbative RG studies (Wetterich 2008, Sengupta & ND 2007)

$$S = \int_0^\beta d\tau \int d^d r \left[ Z |\nabla \psi|^2 + Z_1 \psi^* \partial_\tau \psi + V |\partial_\tau \psi|^2 + r_0 |\psi|^2 + u |\psi|^4 \right]$$

with $Z_1 \rightarrow 0$ and $V > 0$

Same IR behavior as that obtained from the Bose-Hubbard model near the SF-MI transition

The critical behavior of the SF-MI transition in the Bose-Hubbard model appears to be intimately connected to the non-trivial infrared behavior of the SF phase.
Conclusions

- Ultracold atomic gases enables to study strongly-correlated systems in an unprecedentedly controlled manner (microscopic parameters controlled by external fields).
- SF-MI transition observed in atomic gases in optical lattices.
- It appears difficult to understand the excitation spectrum and the critical behavior at the transition from the Bogoliubov theory. The critical behavior appears to be intimately connected to the non-trivial infrared behavior of the superfluid phase as recently studied within the non-perturbation renormalization group.